[CS302-Data Structures] Grade Improvement Homework: Prim's Algorithm

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Section 1. Prim's Algorithm Outline

Prim's Algorithm is type of greedy algorithm that identifies the Minimum Spanning Tree (MST) for a weighted undirected graph. Thus, the Prim's Algorithm finds the subset of the edges of this graph that forms a tree that includes every vertex of the graph, while the total weight of all the edges in that tree gets minimize. The Prim's Algorithm functions by building this tree one vertex at time, starting from an arbitrarily selected first vertex. At each step it adds the cheapest possible connection from the tree to another vertex.

A Basic Description of the Prim's Algorithm is as follows (source: Wikipedia):

1. Initialize a tree with a single vertex, chosen arbitrarily from the graph.
2. Grow the tree by one edge: of the edges that connect the tree to vertices not yet in the tree, find the minimum-weight edge, and transfer it to the tree.
3. Repeat step 2 (until all vertices are in the tree).

In more detail, it may be implemented following the pseudocode below.

1. Associate with each vertex v of the graph a number C[v] (the cheapest cost of a connection to v) and an edge E[v] (the edge providing that cheapest connection). To initialize these values, set all values of C[v] to +∞ (or to any number larger than the maximum edge weight) and set each E[v] to a special flag value indicating that there is no edge connecting v to earlier vertices.
2. Initialize an empty forest F and a set Q of vertices that have not yet been included in F (initially, all vertices).
3. Repeat the following steps until Q is empty:
    a. Find and remove a vertex v from Q having the minimum possible value of C[v]
    b. Add v to F and, if E[v] is not the special flag value, also add E[v] to F
    c. Loop over the edges vw connecting v to other vertices w. For each such edge, if w still belongs to Q and vw has smaller weight than C[w], perform the following steps:
        i. Set C[w] to the cost of edge vw
    d. Set E[w] to point to edge vw.
4. Return F

Section 2. Prim’s Algorithm Programming Assignment

For a weighted undirected graph consisting of 6 vertices implement the Prim’s Algorithm and find the Minimum Spanning Tree. For the implementation of the graph simply use the list of vertices and the Adjacency Matrix formulation. Provide an example code file that creates the graph, calculates the Minimum Spanning Tree and prints it in the following manner:

Edge:	**Weight**
Vertex i – Vertex j	Weight_{ij}

In the report you are also asked to explain *in text* why the result of your algorithm is the MST.