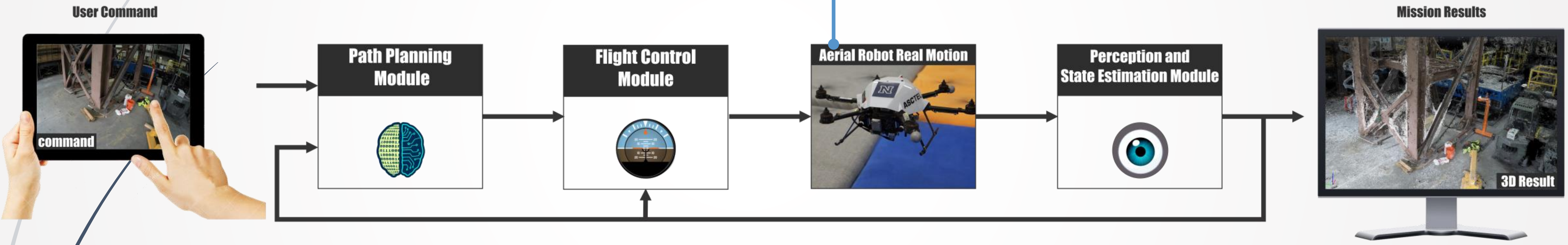
CS491/691: Introduction to Aerial Robotics

Topic: Coordinate Frames

Dr. Kostas Alexis (CSE)

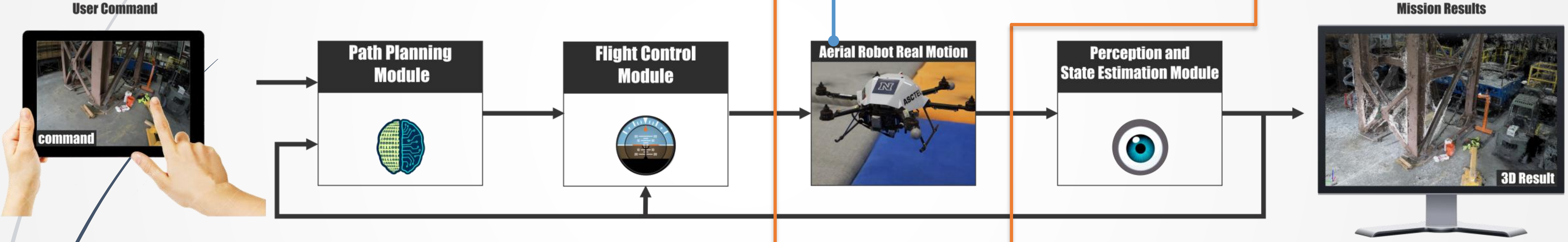
The Aerial Robot Loop

- Real-life Aerial Robot expressing its dynamic behavior in response to the control inputs and external disturbances



The Aerial Robot Loop

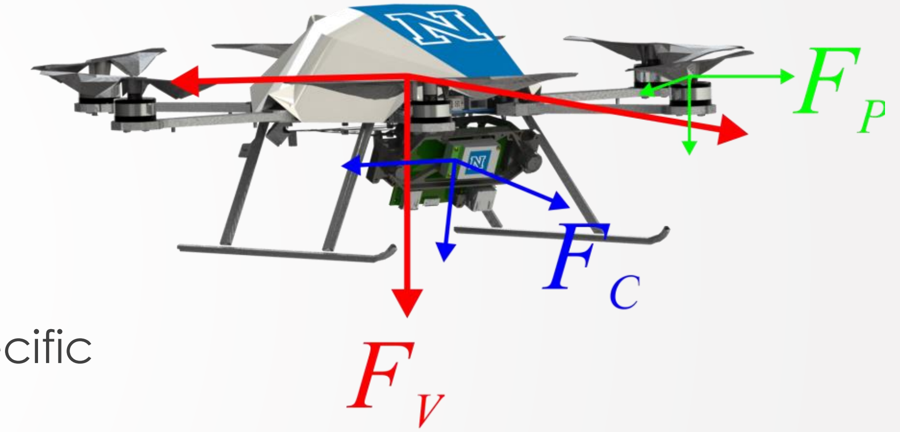
- Real-life Aerial Robot expressing its dynamic behavior in response to the control inputs and external disturbances

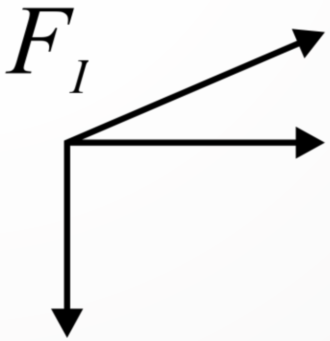


Section 1 of our course

Coordinate Frames

- In Guidance, Navigation and Control of aerial robots, reference coordinate frames are fundamental.
- Describe the relative position and orientation of:
  - Aerial Robot ***relative*** to the Inertial Frame
  - On-board Camera ***relative*** to the Aerial Robot body
  - Aerial Robot ***relative*** to Wind Direction
- Some expressions are easier to formulate in specific frames:
  - Newton's law
  - Aerial Robot Attitude
  - Aerodynamic forces/moments
  - Inertial Sensor data
  - GPS coordinates
  - Camera frames



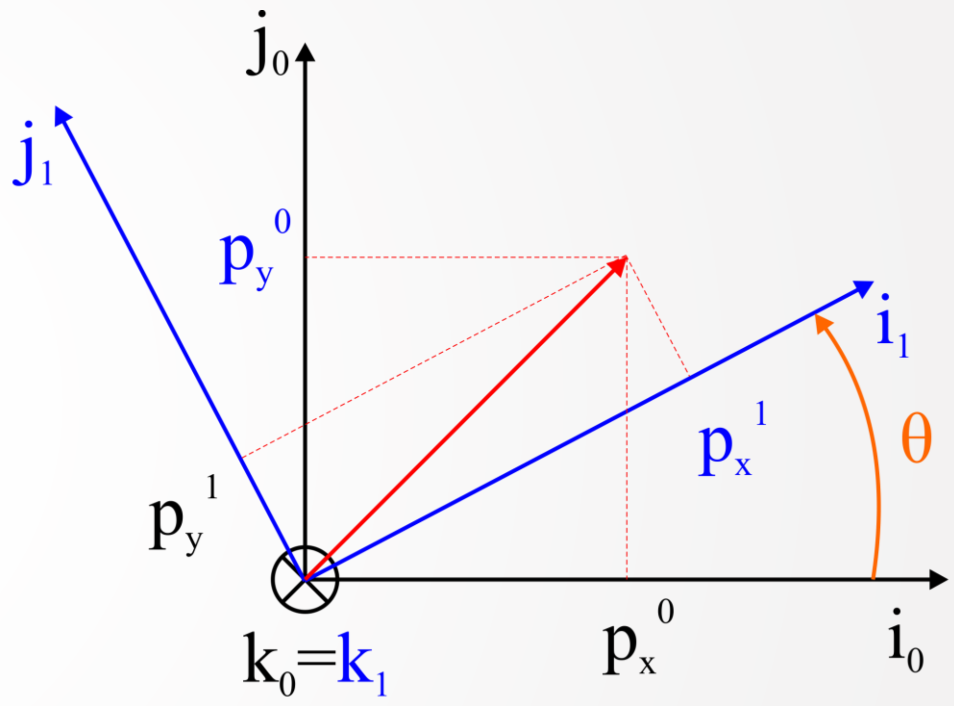


Rotation of Reference Frame

- Rotation around the k-axis

$$\mathbf{p} = p_x^0\mathbf{i}^0 + p_y^0\mathbf{j}^0 + p_z^0\mathbf{k}^0$$
$$\mathbf{p} = p_x^1\mathbf{i}^1 + p_y^1\mathbf{j}^1 + p_z^1\mathbf{k}^1$$
$$\mathbf{p}^1 = \begin{pmatrix} p_x^1 \\ p_y^1 \\ p_z^1 \end{pmatrix} = \begin{pmatrix} \mathbf{i}^1\mathbf{i}^0 & \mathbf{i}^1\mathbf{j}^0 & \mathbf{i}^1\mathbf{k}^0 \\ \mathbf{j}^1\mathbf{i}^0 & \mathbf{j}^1\mathbf{j}^0 & \mathbf{j}^1\mathbf{k}^0 \\ \mathbf{k}^1\mathbf{i}^0 & \mathbf{k}^1\mathbf{j}^0 & \mathbf{k}^1\mathbf{k}^0 \end{pmatrix} \begin{pmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \end{pmatrix}$$

$$\mathbf{p}^1 = \mathcal{R}_0^1\mathbf{p}^0, \ \mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Rotation of Reference Frame

- Rotation around the i-axis

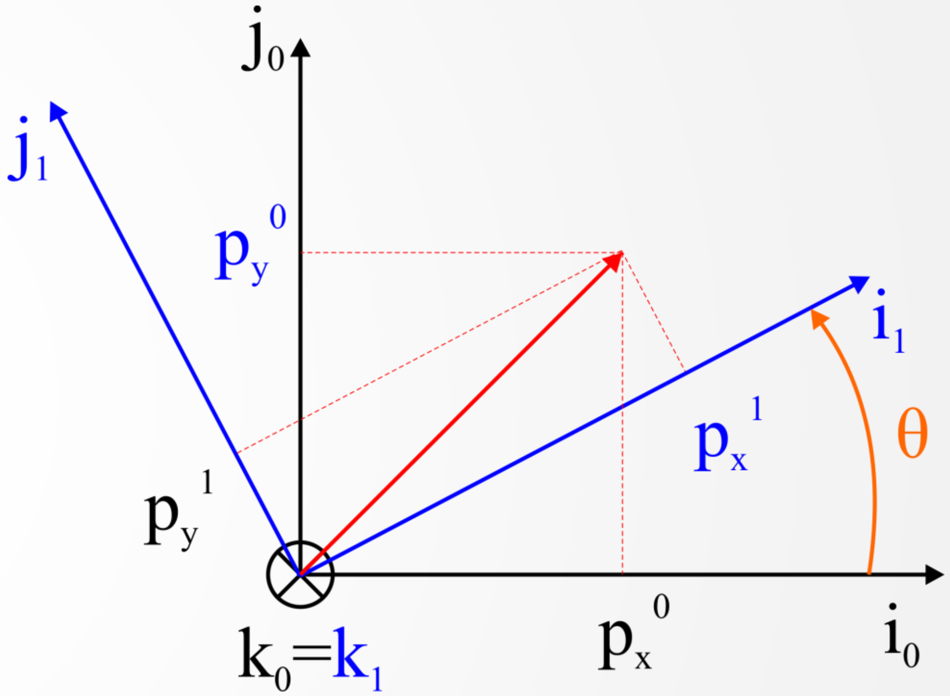
$$\mathcal{R}_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

- Rotation around the j-axis

$$\mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

- Rotation around the k-axis

$$\mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- Orthonormal matrix properties
  - $(\mathcal{R}_a^b)^-1 = (\mathcal{R}_a^b)^T = \mathcal{R}_b^a$
  - $\mathcal{R}_b^c \mathcal{R}a^b = \mathcal{R}_a^c$
  - $\det(\mathcal{R}_a^b) = 1$

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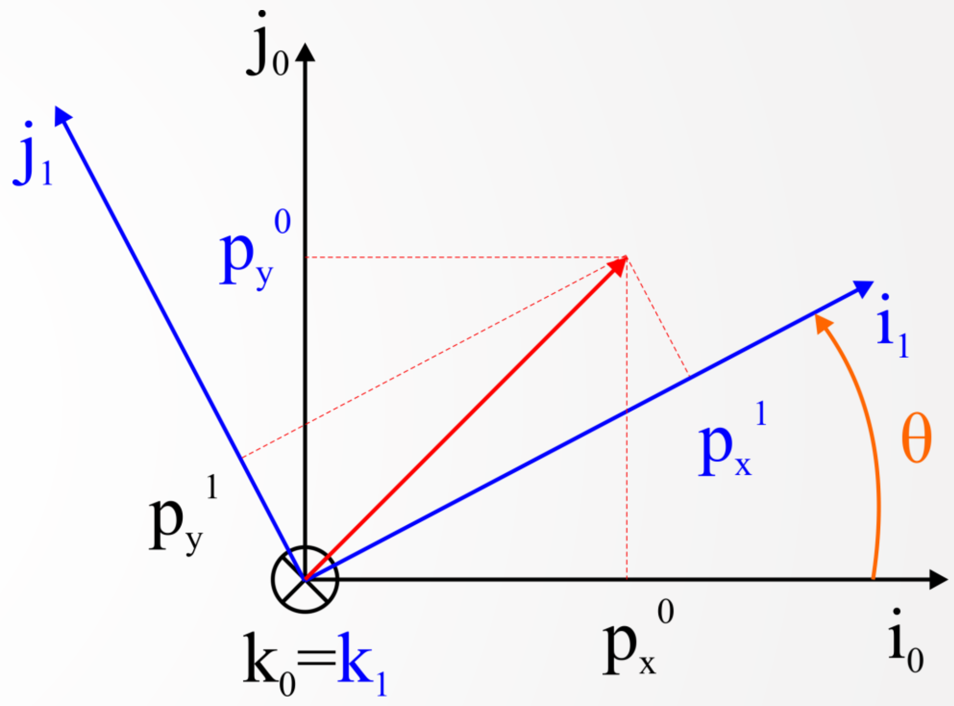

- Orthonormal matrix properties
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  - $\det(\mathcal{R}_a^b) = 1$

Rotation of Reference Frame

- Let $q = |\mathbf{q}|,\ p = |\mathbf{p}|$

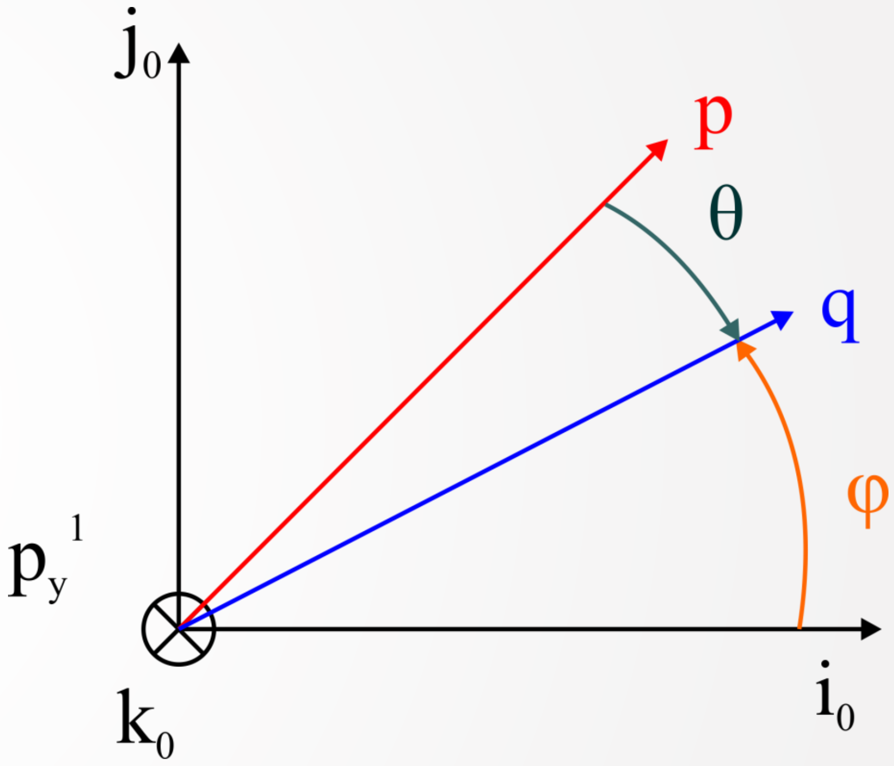
$$\mathbf{p} = \begin{bmatrix} p\cos(\theta+\phi) \\ p\sin(\theta+\phi) \\ 0 \end{bmatrix} = \begin{bmatrix} p\cos\theta\cos\phi - p\sin\theta\sin\phi \\ p\sin\theta\cos\phi + p\cos\theta\sin\phi \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p\cos\phi \\ p\sin\phi \\ 0 \end{bmatrix}$$

- And define:

$$\mathbf{q} = \begin{bmatrix} p\cos\phi \\ p\sin\phi \\ 0 \end{bmatrix}$$

- Then:

$$\mathbf{p} = (\mathcal{R}_0^1)^T\mathbf{q} \Rightarrow \mathbf{q} = \mathcal{R}_0^1\mathbf{p}$$



Rotation of Reference Frame

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$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p\cos\phi \\ p\sin\phi \\ 0 \end{bmatrix}$$

- And define:

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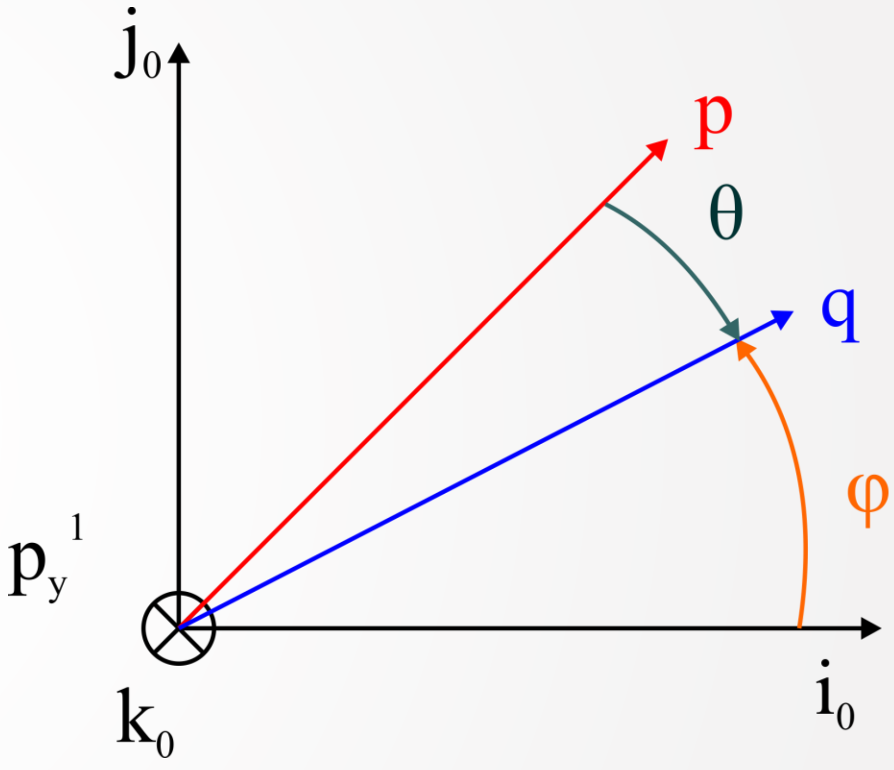
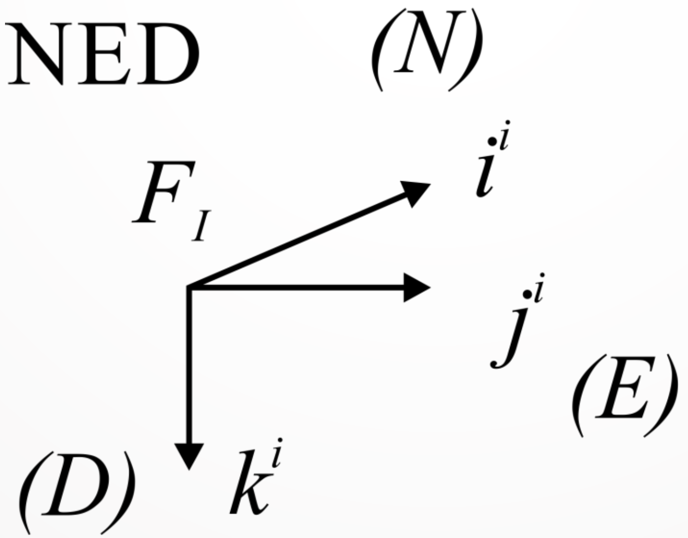
- Then:

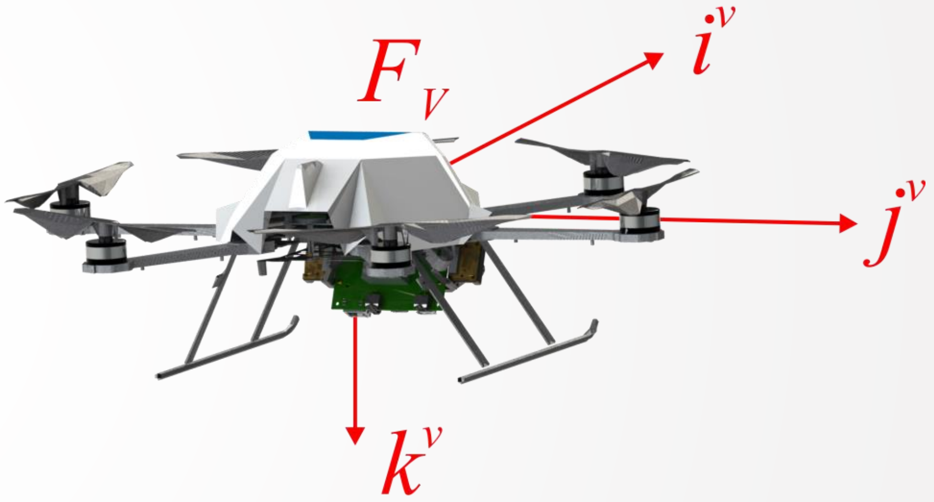
$$\boxed{\mathbf{p} = (\mathcal{R}_0^1)^T\mathbf{q} \Rightarrow \mathbf{q} = \mathcal{R}_0^1\mathbf{p}}$$



Inertial & Vehicle Frames

- Vehicle and Inertial frame have the same orientation.
- Vehicle frame is fixed at the Center of Mass (CoM).
- Both considered as “NED” frames (North-East-Down).





How to represent orientation?

Euler Angles	Quaternions

Euler Angles

$$[\phi, \theta, \psi]$$

- Advantages:
  - Intuitive – directly related with the axis of the vehicle.
- Disadvantages:
  - Singularity – Gimbal Lock.

Quaternions

$$[q_1, q_2, q_3, q_4]$$

- Advantages:
  - Singularity-free.
  - Computationally efficient.
- Disadvantages:
  - Non-intuitive

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We will start here…

ϕ - roll

θ - pitch

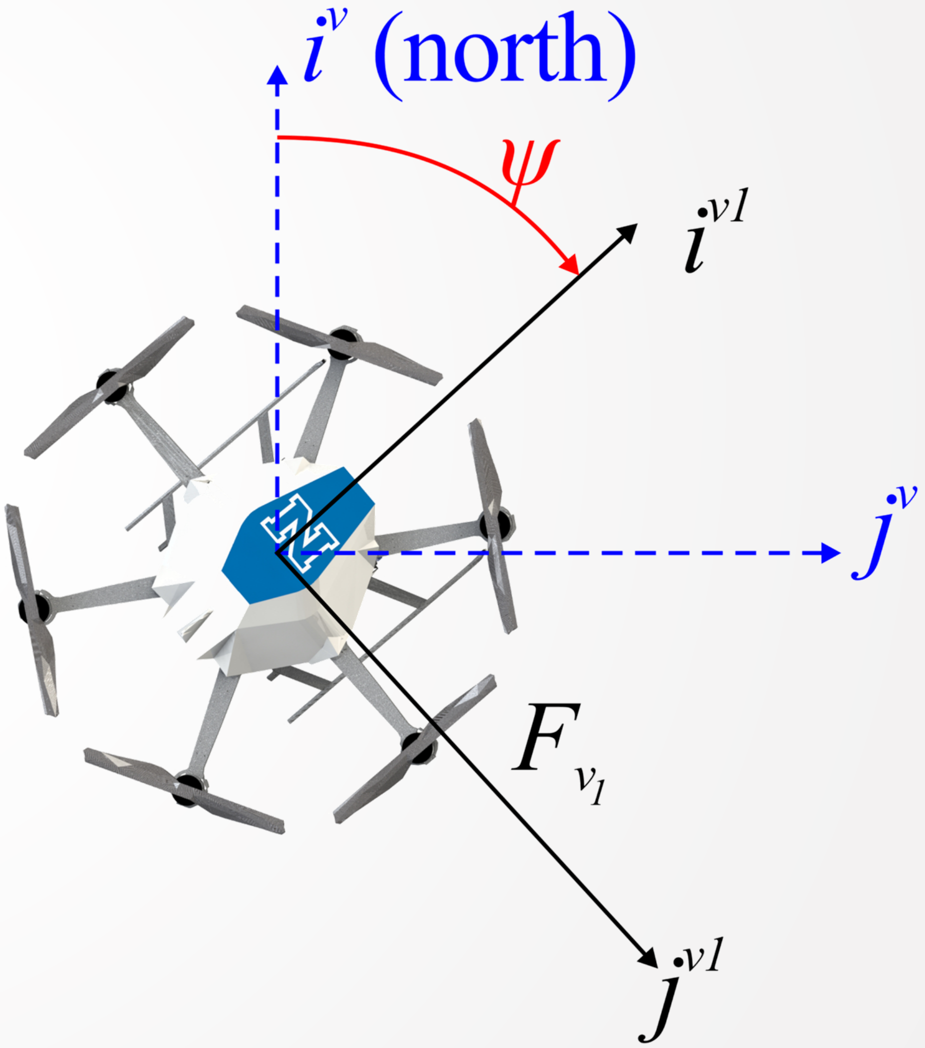
ψ - yaw

Vehicle-1 Frame

$$\mathbf{p}^{v_1} = \mathcal{R}_v^{v_1}\mathbf{p}^v,$$

$$\mathcal{R}_v^{v_1} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ψ represents the yaw angle

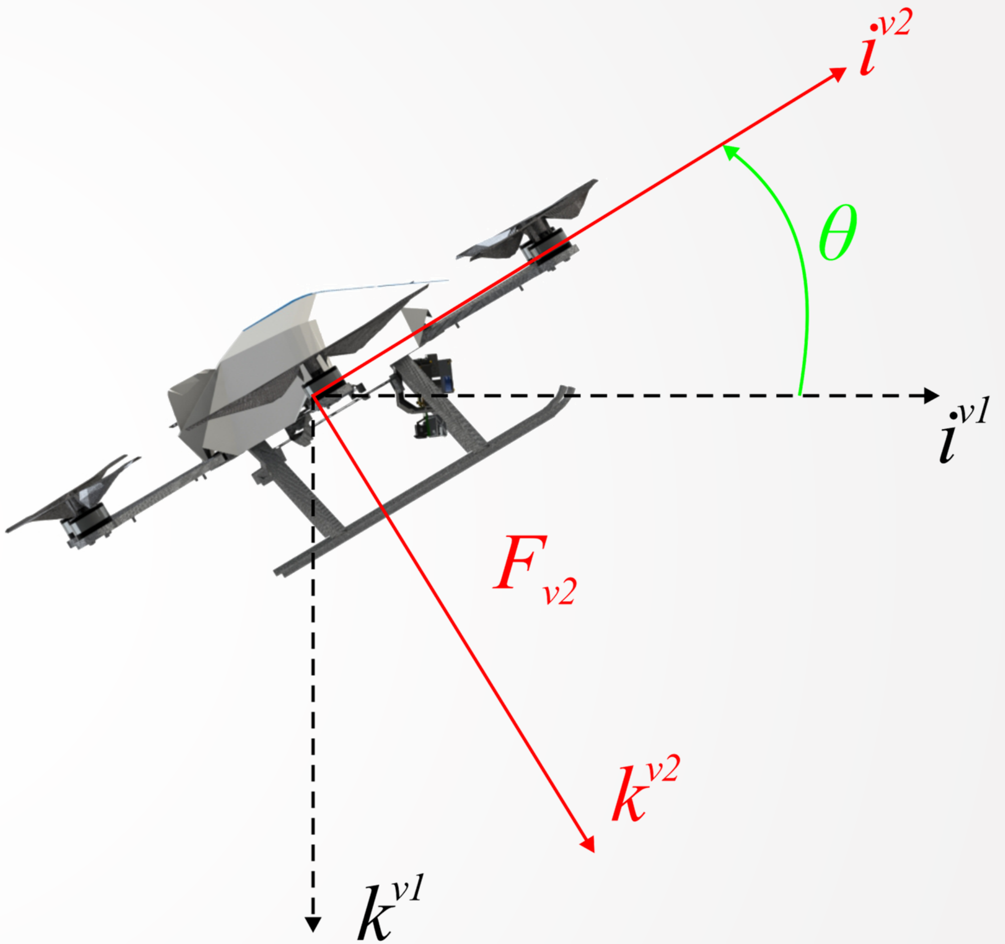


Vehicle-2 Frame

$$\mathbf{p}^{v_2} = \mathcal{R}_{v_1}^{v_2}\mathbf{p}^{v_1},$$

$$\mathcal{R}_{v_1}^{v_2} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

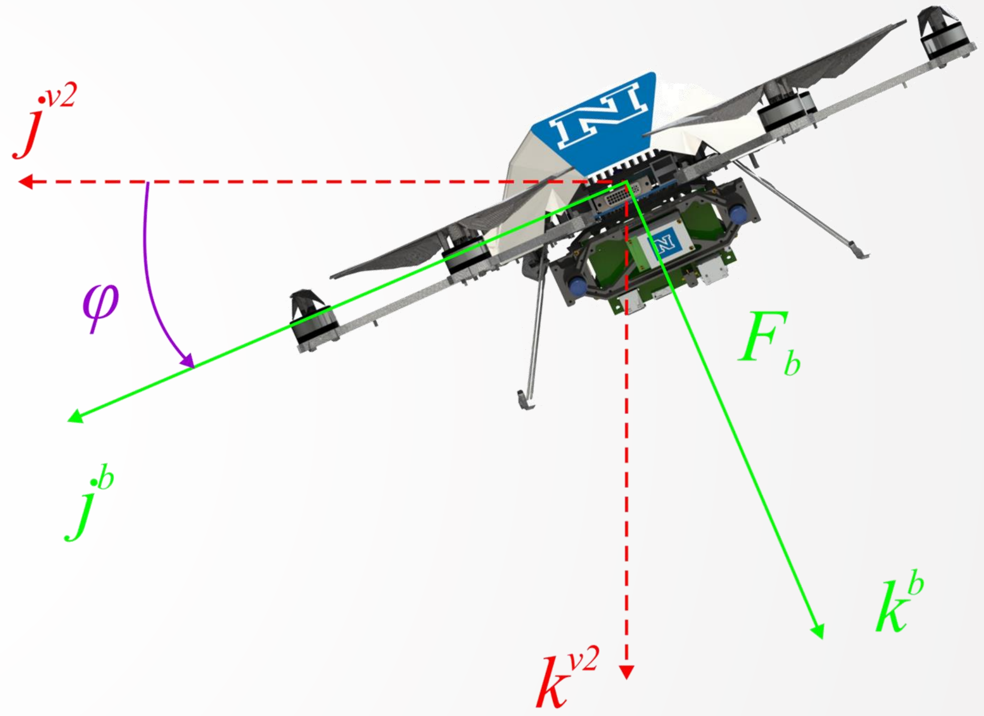
- θ represents the pitch angle



Body Frame

$$\mathbf{p}^b = \mathcal{R}_{v_2}^b \mathbf{p}^{v_2},$$

$$\mathcal{R}_{v_2}^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$



- φ represents the roll angle

Inertial Frame to Body Frame

- Let:

$$\mathcal{R}_v^b(\phi,\theta,\psi) = \mathcal{R}_{v_2}(\phi)\mathcal{R}_{v_1}^{v_2}(\theta)\mathcal{R}_v^{v_1}(\psi)$$

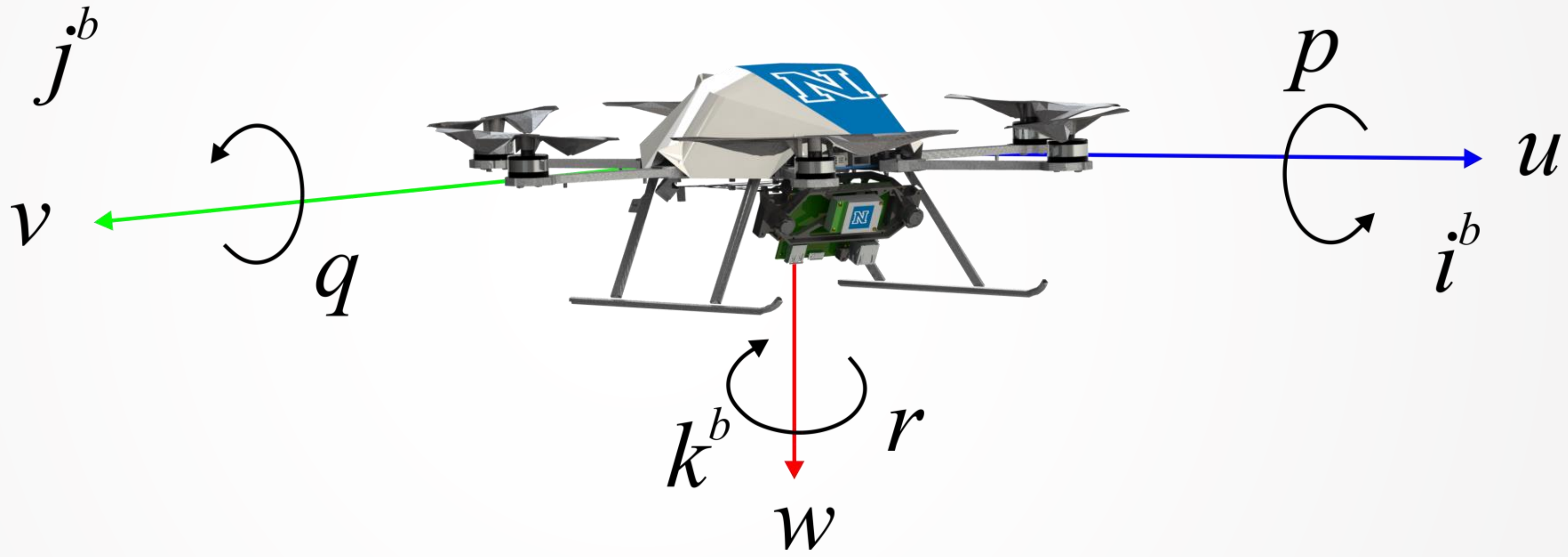
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix}$$

- Then:

$$\mathbf{p}^b = \mathcal{R}_v^b \mathbf{p}^v$$

Further Application to Robot Kinematics



- $[p,q,r]$: body angular rates
- $[u,v,w]$: body linear velocities

Relate Translational Velocity-Position

- Let *[u,v,w]* represent the body linear velocities

$$\frac{d}{dt}\begin{bmatrix} p_n \\ p_e \\ p_d \end{bmatrix} = \mathcal{R}_b^v \begin{bmatrix} u \\ v \\ w \end{bmatrix} = (\mathcal{R}_v^b)^T \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- Which gives:

$$\begin{bmatrix} \dot{p}_n \\ \dot{p}_e \\ \dot{p}_d \end{bmatrix} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\phi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_{c\psi} \\ -s_\theta & s_\phi c_\theta & c_{c\theta} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Body Rates – Euler Rates

- Let *[p,q,r]* denote the body angular rates

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \mathcal{R}_{v_2}^{b}(\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \mathcal{R}_{v_2}^{b}(\phi)\mathcal{R}_{v_1}^{v_2}(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} =$$

$$\begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

- Inverting this expression:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

How to represent orientation?

Euler Angles	Quaternions

Euler Angles

$$[\phi, \theta, \psi]$$

- Advantages:
  - Intuitive – directly related with the axis of the vehicle.
- Disadvantages:
  - Singularity – Gimbal Lock.

Quaternions

$$[q_1, q_2, q_3, q_4]$$

- Advantages:
  - Singularity-free.
  - Computationally efficient.
- Disadvantages:
  - Non-intuitive

A glimpse…

Quaternions

- Complex numbers form a plane : their operations are highly related with 2-dimensional geometry.
- In particular, multiplication by a unit complex number:

$$|z^2| = 1$$

which can all be written:

$$z = e^{i\theta}$$

gives a rotation

$$\mathcal{R}_z(w) = zw$$

by angle θ

Quaternions

- Theorem by Euler states that any given sequence of rotations can be represented as a single rotation about a *fixed-axis*
- Quaternions provide a convenient parametrization of this effective axis and a rotation angle:

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \bar{E} \sin \frac{\zeta}{2} \\ \cos \frac{\zeta}{2} \end{bmatrix}$$

- Where $\bar{E}$ is a unit vector and ζ is a positive rotation about $\bar{E}$

Quaternions

- Note that $|\bar{q}| = 1$ and therefore there are only 3 degrees of freedom in this formulation also.
- If $\bar{q}$ represents the rotational transformation from the reference frame A to the reference frame B, the frame A is aligned with B when frame A is rotated by ζ radians around $\bar{E}$
- This representation is connected with the Euler angles form, according to the following expression:

$$\begin{bmatrix} \sin\theta \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} -2(q_2q_4 + q_1q_3) \\ \arctan 2[2(q_2q_3 - q_1q_4), 1 - 2(q_1^2 + q_2^2)] \\ \arctan 2[2(q_1q_2 - q_3q_4), 1 - 2(q_2^2 + q_3^2)] \end{bmatrix}$$

Quaternions

- This representation has the great **advantage** of being:
  - Singularity-free and
  - Computationally efficient to do state propagation (typically within an Extended Kalman Filter)
- On the other hand, it has one main **disadvantage**, namely being far less intuitive.

Euler to Quaternions in Python

```python
#       __QUATEULERMAIN__
#       This main file demonstrates functions for handling
#       and manipulating quaternios and Euler Angles
#
#       Authors:
#       Kostas Alexis (kalexis@unr.edu)

from numpy import *
import numpy as np
from QuatEulerFunctions import *

# demo values
q_ = np.array([0.25,0.5,0.1,0.2])

print 'Quaternion: '
print q_

rpy_ = quat2rpy(q_)
print 'Euler angles'
print rpy_

quat_ = rpy2quat(rpy_)
print 'Recovered quaternion:'
print quat_

rot_ = quat2r(quat_)
print 'Recovered rotation matrix:'
print rot_

rpy_rec_ = r2rpy(rot_)
print 'Recovered Euler'
print rpy_rec_

q_norm_ = normalized(q_)
print 'Normalized quaternion:'
print q_norm_
```

```python
#       __QUATEULERFUNCTIONS__
#       This file implements functions for handling
#       and manipulating quaternios and Euler Angles
#
#       Authors:
#       Kostas Alexis (kalexis@unr.edu)

from numpy import *
import numpy as np

def quat2r(q_AB=None):
    C_AB = np.zeros((3,3))
    C_AB[0,0] = q_AB[0]*q_AB[0] - q_AB[1]*q_AB[1] - q_AB[2]*q_AB[2] + q_AB[3]*q_AB[3]
    C_AB[0,1] = q_AB[0]*q_AB[1]*2.0 + q_AB[2]*q_AB[3]*2.0
    C_AB[0,2] = q_AB[0]*q_AB[2]*2.0 - q_AB[1]*q_AB[3]*2.0

    C_AB[1,0] = q_AB[0]*q_AB[1]*2.0 - q_AB[2]*q_AB[3]*2.0
    C_AB[1,1] = -q_AB[0]*q_AB[0] + q_AB[1]*q_AB[1] - q_AB[2]*q_AB[2] + q_AB[3]*q_AB[3]
    C_AB[1,2] = q_AB[0]*q_AB[3]*2.0 - q_AB[1]*q_AB[2]*2.0

    C_AB[2,0] = q_AB[0]*q_AB[2]*2.0 + q_AB[1]*q_AB[3]*2.0
    C_AB[2,1] = q_AB[0]*q_AB[3]*(-2.0) + q_AB[1]*q_AB[2]*2.0
    C_AB[2,2] = -q_AB[0]*q_AB[0] - q_AB[1]*q_AB[1] + q_AB[2]*q_AB[2] + q_AB[3]*q_AB[3]
    return C_AB

def quat2rpy(q_AB=None):
    C = quat2r(q_AB)
    theta = np.arcsin(-C[2,0])
    phi = np.arctan2(C[2,1],C[2,2])
    psi = np.arctan2(C[1,0],C[0,0])
    rpy = np.zeros((3,1))
    rpy[0] = phi
    rpy[1] = theta
    rpy[2] = psi
    return rpy

def rpy2quat(rpy=None):
    r = rpy[0]
    p = rpy[1]
    y = rpy[2]
    cRh = np.cos(r/2)
    sRh = np.sin(r/2)
    cPh = np.cos(p/2)
    sPh = np.sin(p/2)
    cYh = np.cos(y/2)
    sYh = np.sin(y/2)
    qs_cmpl = np.array([ -(np.multiply(np.multiply(sRh,cPh),cYh) - np.multiply(np.multiply(cRh,sPh),sYh)),
                        -(np.multiply(np.multiply(cRh,sPh),cYh) + np.multiply(np.multiply(sRh,cPh),sYh)),
                        -(np.multiply(np.multiply(cRh,cPh),sYh) - np.multiply(np.multiply(sRh,sPh),cYh)),
                        np.multiply(np.multiply(cRh,cPh),cYh) + np.multiply(np.multiply(sRh,sPh),sYh)])
    qs = np.real(qs_cmpl)
    return qs

def r2rpy(C=None):
    theta = np.arcsin(-C[2,0])
    phi = np.arctan2(C[2,1],C[2,2])
    psi = np.arctan2(C[1,0],C[0,0])
    rpy = np.zeros((3,1))
    rpy[0] = phi
    rpy[1] = theta
    rpy[2] = psi
    return rpy

def normalized(x=None):
    y=x/np.sqrt(np.dot(x,x))
    return y
```

Euler to Quaternions in Python

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    C_AB[2,2] = -q_AB[0]*q_AB[0] - q_AB[1]*q_AB[1] + q_AB[2]*q_AB[2] + q_AB[3]*q_AB[3]
    return C_AB

def quat2rpy(q_AB=None):
    C = quat2r(q_AB)
    theta = np.arcsin(-C[2,0])
    phi = np.arctan2(C[2,1],C[2,2])
    psi = np.arctan2(C[1,0],C[0,0])
    rpy = np.zeros((3,1))
    rpy[0] = phi
    rpy[1] = theta
    rpy[2] = psi
    return rpy
```

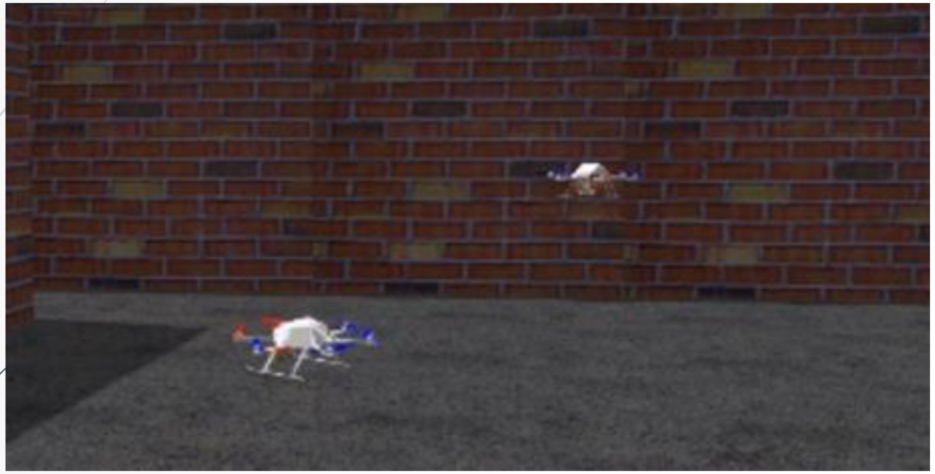
```python
def normalized(x=None):
    y=x/np.sqrt(np.dot(x,x))
    return y
```

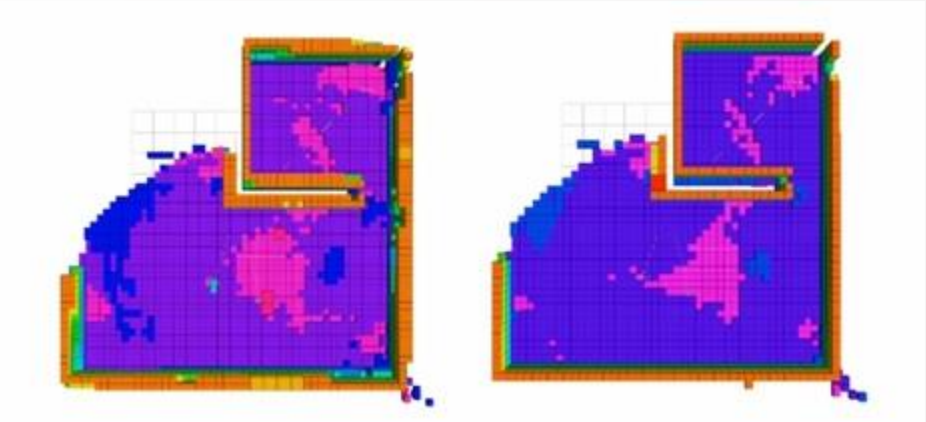
Flight Training

- Realistic Flight Simulation and RC Interface for Safety Piloting.
- Available at our lab at the Applied Research Facilities – Rooms 118-166
- Contact initially me and subsequently we will make a google calendar

- Goal:
  - Enable you with flying skills.
  - Attract/Support Aerial Robotics Research at UNR.

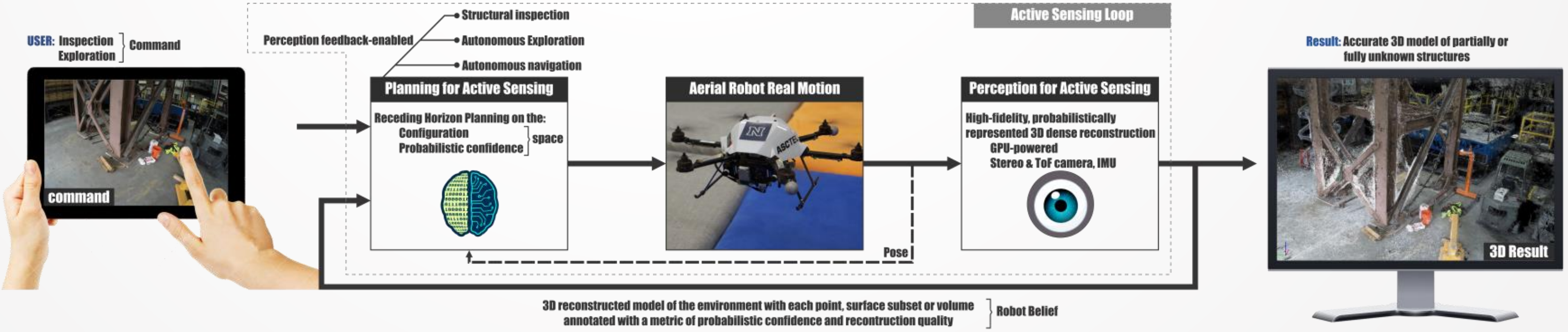
Development Framework: RotorS





- RotorS is a MAV gazebo simulator developed by the Autonomous Systems Lab at ETH Zurich. It provides some multirotor models such as the AscTec Hummingbird, the AscTec Pelican, or the AscTec Firefly. There are simulated sensors coming with the simulator such as an IMU, a generic odometry sensor, and a Visual-Inertial sensor, which can be mounted on the multirotor. This package also contains some example controllers, basic worlds, a joystick interface, and example launch files. Below we provide the instructions necessary for getting started. See RotorS' wiki for more instructions and examples (https://github.com/ethz-asl/rotors_simulator/wiki).
- Pre-compiled version on Ubuntu Virtual Machine: http://www.kostasalexis.com/rotors-simulator1.html

Research Section



Find out more

- http://www.kostasalexis.com/frame-rotations-and-representations.html
- http://page.math.tu-berlin.de/~plaue/plaue_intro_quats.pdf
- http://mathworld.wolfram.com/RotationMatrix.html
- http://mathworld.wolfram.com/EulerAngles.html
- http://blog.wolframalpha.com/2011/08/25/quaternion-properties-and-interactive-rotations-with-wolframalpha/
- http://www.mathworks.com/discovery/rotation-matrix.html
- http://www.mathworks.com/discovery/quaternion.html?refresh=true
- http://www.cprogramming.com/tutorial/3d/rotationMatrices.html
- http://www.cprogramming.com/tutorial/3d/quaternions.html

- **Help with Linear Algebra?** https://www.khanacademy.org/math/linear-algebra
- **Always check:** http://www.kostasalexis.com/literature-and-links.html

Transpose of 3x3 Matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Determinant of 3x3 Matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\det \mathbf{A} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Thank you!

Please ask your question!