

CS491/691: Introduction to Aerial Robotics

Topic: Short Recap of Lecture 2

Dr. Kostas Alexis (CSE)

Recap from previous

• Rotation around the k-axis $\mathbf{p} = p_x^0 \mathbf{i}^0 + p_y^0 \mathbf{j}^0 + p_z^0 \mathbf{k}^0$ $\mathbf{p} = p_x^1 \mathbf{i}^1 + p_y^1 \mathbf{j}^1 + p_z^1 \mathbf{k}^1$ $\mathbf{p}^1 = \begin{pmatrix} p_x^1 \\ p_y^1 \\ p_z^1 \end{pmatrix} = \begin{pmatrix} \mathbf{i}^1 \mathbf{i}^0 & \mathbf{i}^1 \mathbf{j}^0 & \mathbf{i}^1 \mathbf{k}^0 \\ \mathbf{j}^1 \mathbf{i}^0 & \mathbf{j}^1 \mathbf{j}^0 & \mathbf{j}^1 \mathbf{k}^0 \\ \mathbf{k}^1 \mathbf{i}^0 & \mathbf{k}^1 \mathbf{j}^0 & \mathbf{k}^1 \mathbf{k}^0 \end{pmatrix} \begin{pmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \end{pmatrix}$

$$\mathbf{p}^{1} = \mathcal{R}_{0}^{1} \mathbf{p}^{0}, \ \mathcal{R}_{0}^{1} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$







$$\mathbf{p}^{v_2} = \mathcal{R}_{v_1}^{v_2} \mathbf{p}^{v_1},$$
$$\mathcal{R}_{v_1}^{v_2} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

• θ represents the pitch angle





Inertial Frame to Body Frame

$$\begin{array}{l} \text{Let:} \\ \mathcal{R}_{v}^{b}(\phi,\theta,\psi) = \mathcal{R}_{v_{2}}(\phi)\mathcal{R}_{v_{1}}^{v_{2}}(\theta)\mathcal{R}_{v}^{v_{1}}(\psi) \\ \\ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \\ = \begin{bmatrix} c_{\theta}c_{\psi} & c_{\theta}s_{\psi} & -s_{\theta} \\ s_{\phi}s_{\theta}c_{\psi} - c_{\phi}s_{\psi} & s_{\phi}s_{\theta}s_{\psi} + c_{\phi}c_{\psi} & s_{\phi}c_{\theta} \\ c_{\phi}s_{\theta}c_{\psi} + s_{\phi}s_{\psi} & c_{\phi}s_{\theta}s_{\psi} - s_{\phi}c_{\psi} & c_{\phi}c_{\theta} \end{bmatrix}$$

Then:

$$\mathbf{p}^b = \mathcal{R}^b_v \mathbf{p}^v$$

Rotation of Reference Frame

Rotation around the i-axis

 $\mathcal{R}_0^1 = \begin{bmatrix} 1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

Rotation around the j-axis

$$\mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

Rotation around the k-axis

$$\mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1 \end{bmatrix}$$



Orthonormal matrix properties

•
$$(\mathcal{R}^b_a)^- 1 = (\mathcal{R}^b_a)^T = \mathcal{R}^b_a$$

$$\bullet \ \mathcal{R}_b^c \mathcal{R} a^b = \mathcal{R}_a^c$$

•
$$\det(\mathcal{R}^b_a) = 1$$

Further Application to Robot Kinematics



- [p,q,r]: body angular rates
- [u,v,w]: body linear velocities

Relate Translational Velocity-Position

Let [u,v,w] represent the body linear velocities

$$\frac{d}{dt} \begin{bmatrix} p_n \\ p_e \\ p_d \end{bmatrix} = \mathcal{R}_b^v \begin{bmatrix} u \\ v \\ w \end{bmatrix} = (\mathcal{R}_v^b)^T \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Which gives:

$$\begin{bmatrix} \dot{p}_n \\ \dot{p}_e \\ \dot{p}_d \end{bmatrix} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\phi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_{c\psi} \\ -s_\theta & s_\phi c_\theta & c_{c\theta} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Body Rates – Euler Rates

Let [p,q,r] denote the body angular rates

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \mathcal{R}_{v_2}^{b}(\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \\ 0 \end{bmatrix} + \mathcal{R}_{v_2}^{b}(\phi) \mathcal{R}_{v_1}^{v_2}(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi & \cos\theta \\ 0 & -\sin\phi & \cos\phi & \cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Inverting this expression:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Thank you! Rlease ask your question! General and anness

日日