Drones Demystified!

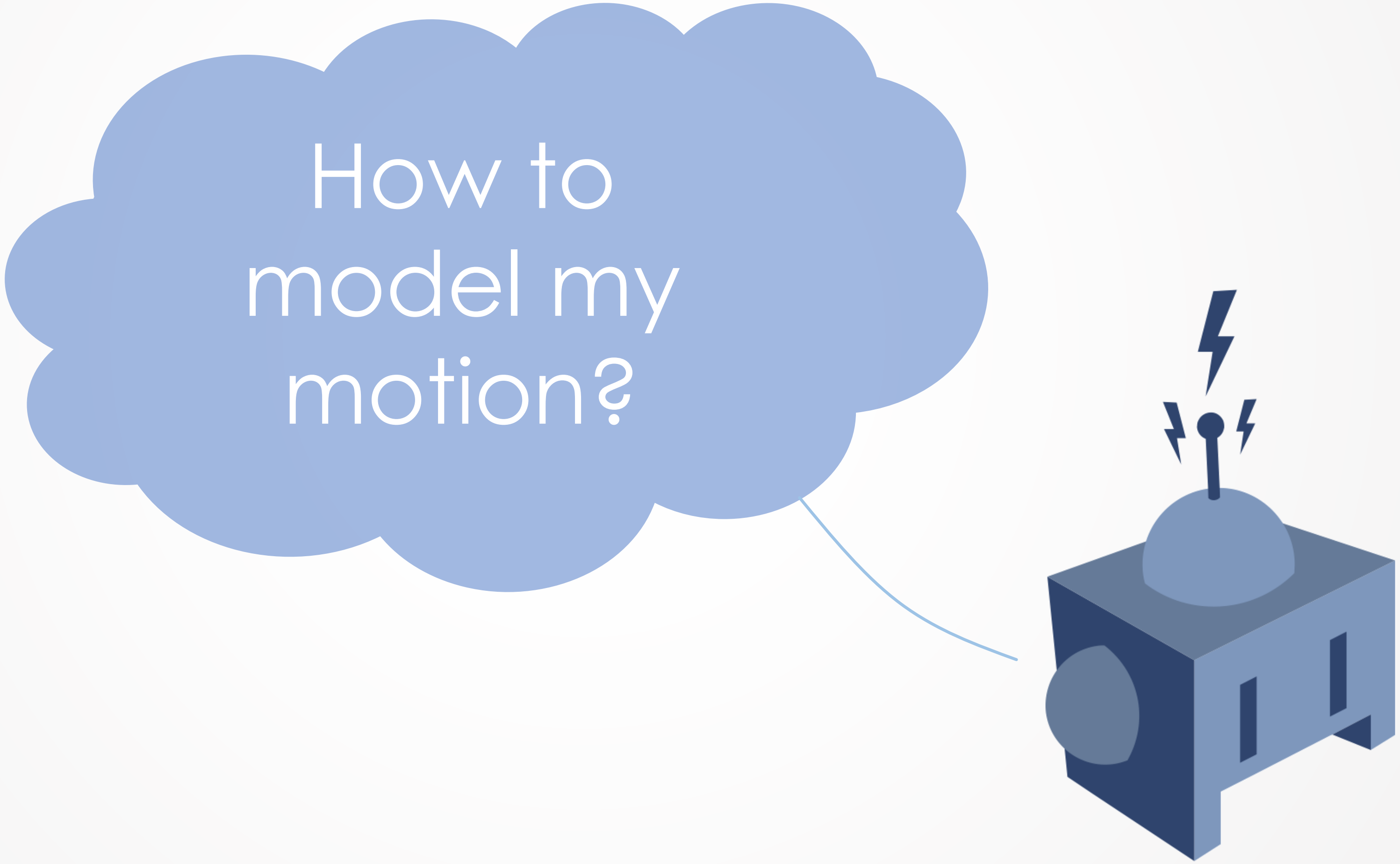
K. Alexis, C. Papachristos, Autonomous Robots Lab, University of Nevada, Reno

A. Tzes, Autonomous Robots & Intelligent Systems Lab, NYU Abu Dhabi

Drones Demystified!

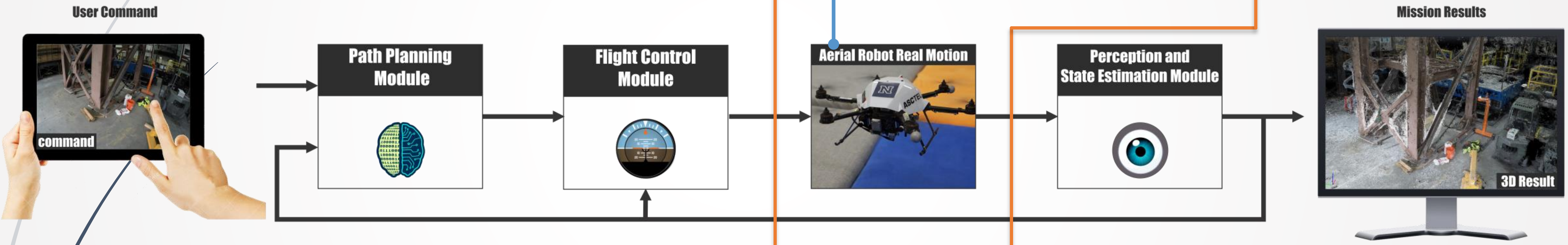
Topic: Micro Aerial Vehicle Dynamics

Propulsion Systems for Robotics



The Aerial Robot Loop

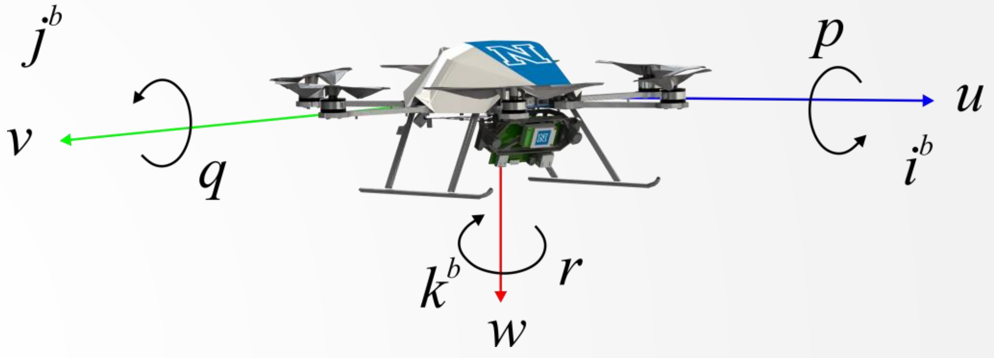
- Real-life Aerial Robot expressing its dynamic behavior in response to the control inputs and external disturbances



Section 1 of our course

Goal of this lecture

- The goal of this lecture is to derive the equations of motion that describe the motion of a multirotor Micro Aerial Vehicle.
- The MAV has 6 Degrees of Freedom but only 4 distinct inputs.
  - It is an underactuated system.
- To achieve this goal, we rely on:
  - A model of the Aerodynamic Forces & Moments
  - A model of the motion of the vehicle body as actuated by the forces and moments acting on it.



The MAV Propeller

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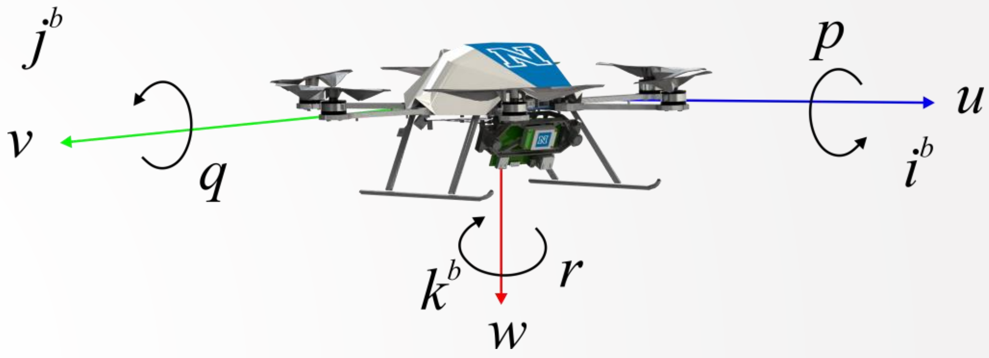
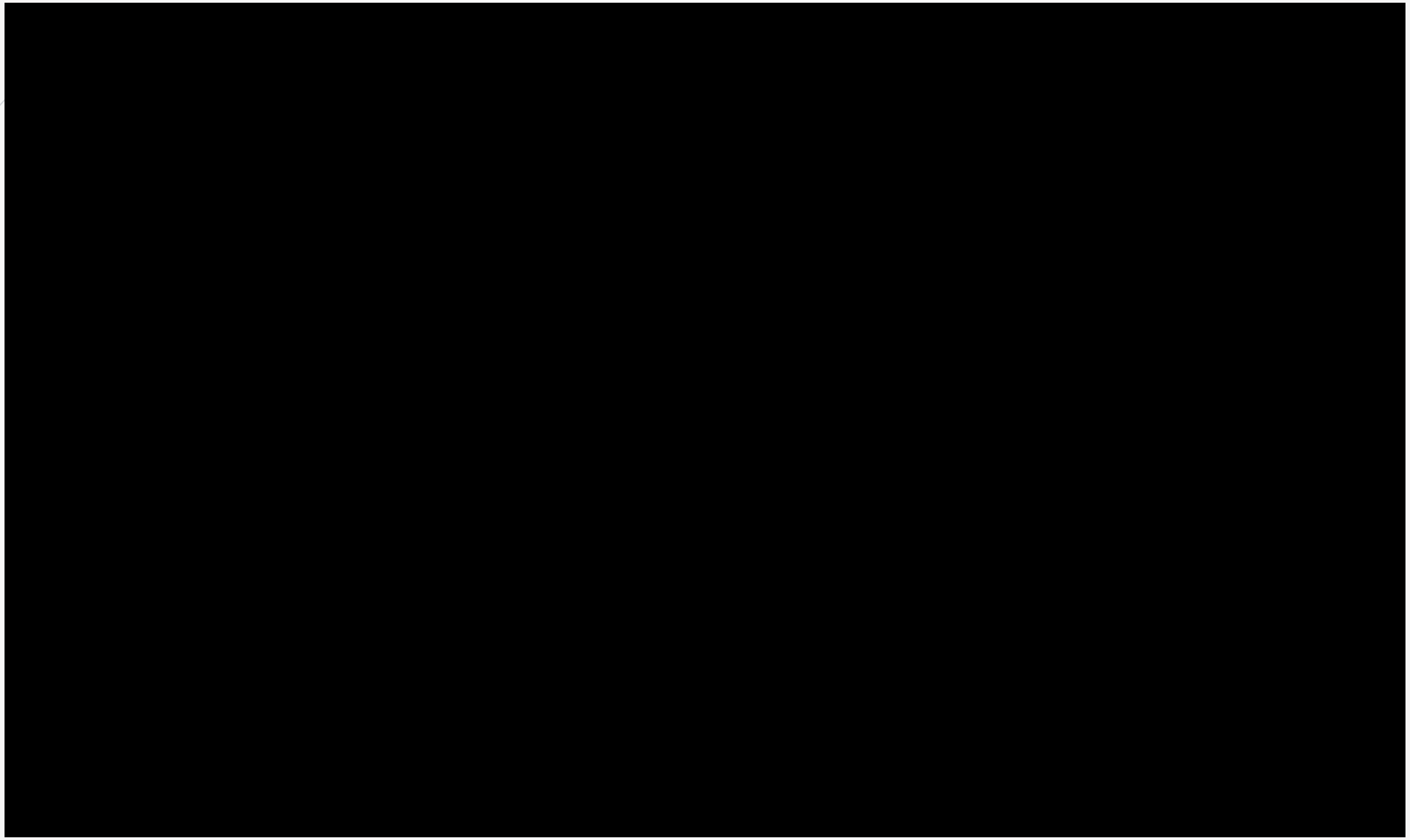


The MAV Propeller

- **Is something much simpler than a helicopter rotor**

The MAV Propeller



- Video of airflow and vortex patterns with propellers. These tests were conducted at NACA, now NASA Langley Research Center. The interior tests were probably at the Propeller Research Tunnel. The exterior tests at the end of the film were at the Helicopter Test Tower. Langley Film #L-118

The MAV Propeller

- Rotor modeling is a very complicated process.
- **A Rotor is different than a propeller.** It is not-rigid and contains degrees of freedom. Among them blade flapping allows the control of the rotor tip path plane and therefore control the helicopter.

- Used to produce thrust.
- Propeller plane perpendicular to shaft.
- Rigid blade. No flapping.
- Fixed blade pitch angle or collective changes only.

- Used to produce lift and directional control.
- Elastic element between blade and shaft.
- Blade flapping used to change tip path plane.
- Blade pitch angle controlled by swashplate.

The MAV Propeller

- Simplified model forces and moments:
  - **Thrust Force:** the resultant of the vertical forces acting on all the blade elements.

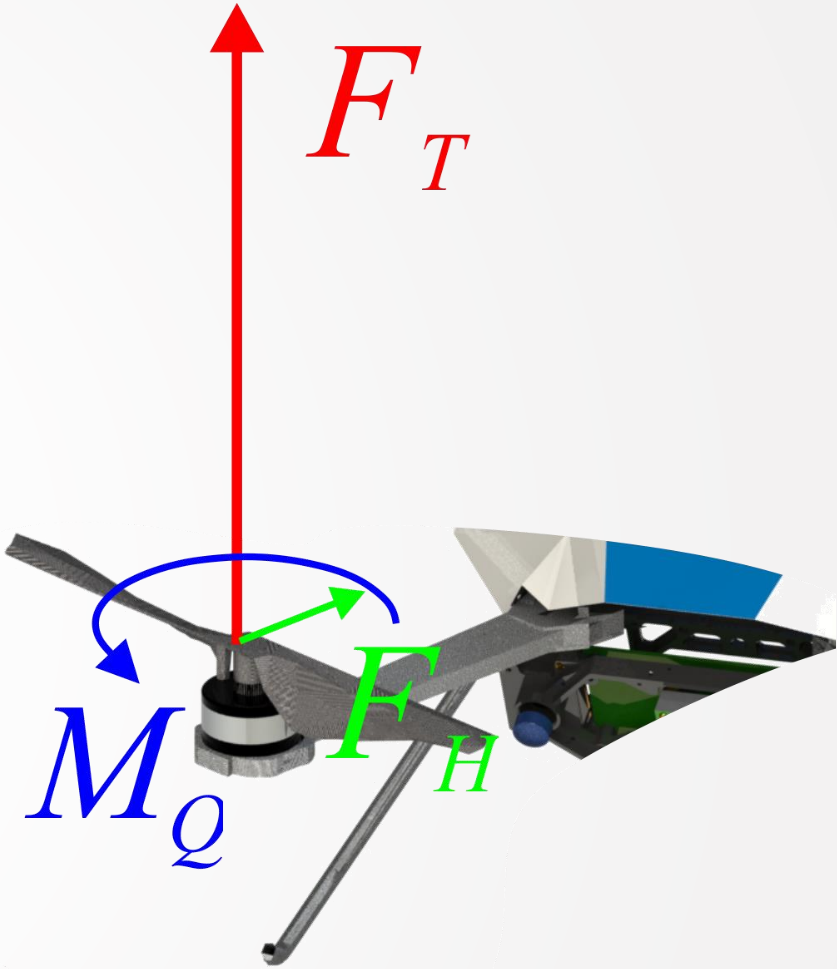
$$F_T = T = C_T \rho A (\Omega R)^2$$

  - **Hub Force:** the resultant of all the horizontal forces acting on all the blade elements.

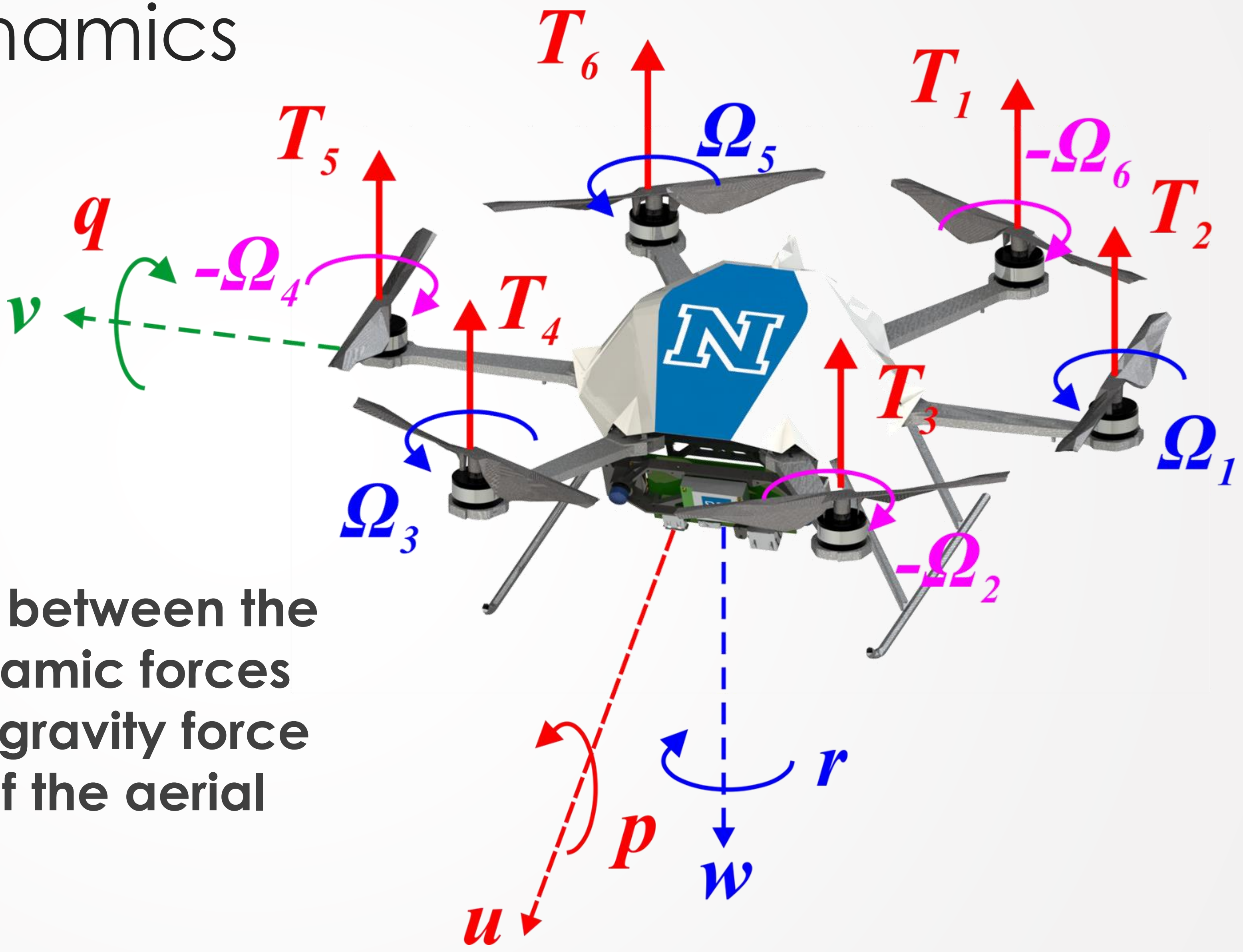
$$F_H = H = C_H \rho A (\Omega R)^2$$

  - **Drag Moment:** This moment about the rotor shaft is caused by the aerodynamic forces acting on the blade elements. The horizontal forces acting on the rotor are multiplied by the moment arm and integrated over the rotor. Drag moment determines the power required to spin the rotor.

$$M_Q = Q = C_Q \rho A (\Omega R)^2 R$$

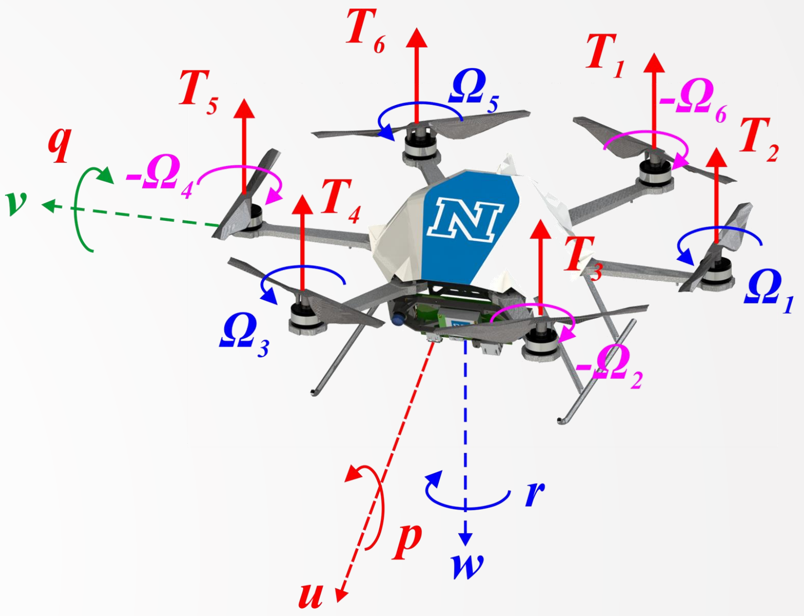


MAV Dynamics



What is the relation between the propeller aerodynamic forces and moments, the gravity force and the motion of the aerial robot?

MAV Dynamics



- Assumption 1: the Micro Aerial Vehicle is flying as a rigid body with negligible aerodynamic effects on it – for the employed airspeeds.
- The propeller is considered as a simple propeller disc that generates thrust and a moment around its shaft.
- Recall:

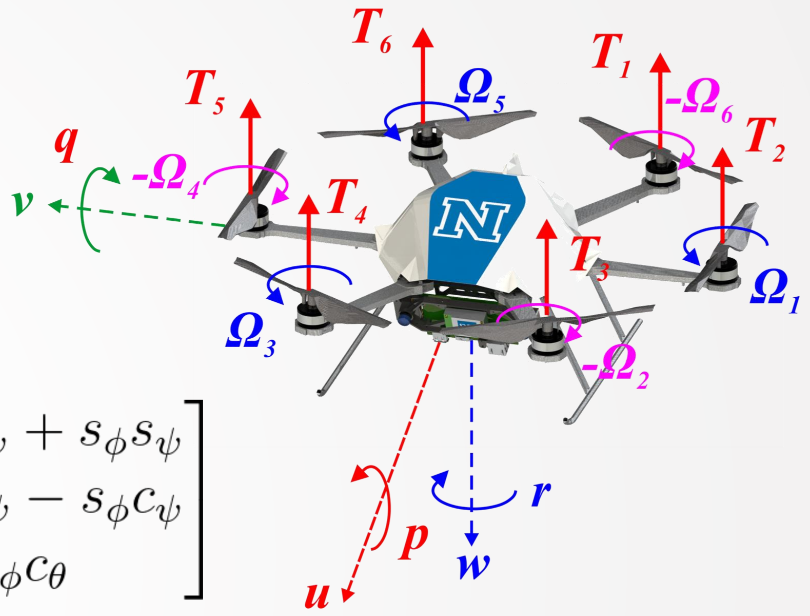
$$F_T = T = C_T \rho A (\Omega R)^2$$
$$M_Q = Q = C_Q \rho A (\Omega R)^2 R$$

- And let us write:

$$T_i = k_n \Omega_i^2$$
$$M_i = (-1)^{i-1} k_m T_i$$

MAV Dynamics

- Recall the kinematic equations:



  - Translational Kinematic Expression:

$$\begin{bmatrix} \dot{p}_n \\ \dot{p}_e \\ \dot{p}_d \end{bmatrix} = \mathcal{R}_b^v \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \mathcal{R}_b^v = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}$$

  - Rotational Kinematic Expression

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

MAV Dynamics

- Recall Newton's 2nd Law:

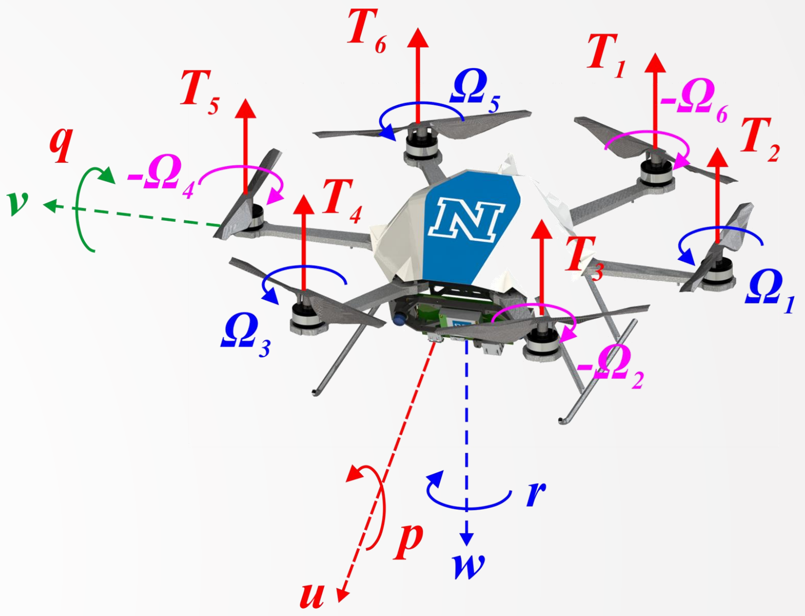
$$m\frac{d\mathbf{V}_g}{dt_i} = \mathbf{f}$$

  - f is the summary of all external forces
  - m is the mass of the robot
  - Time derivative is taken wrt the interial frame
- Using the expression:

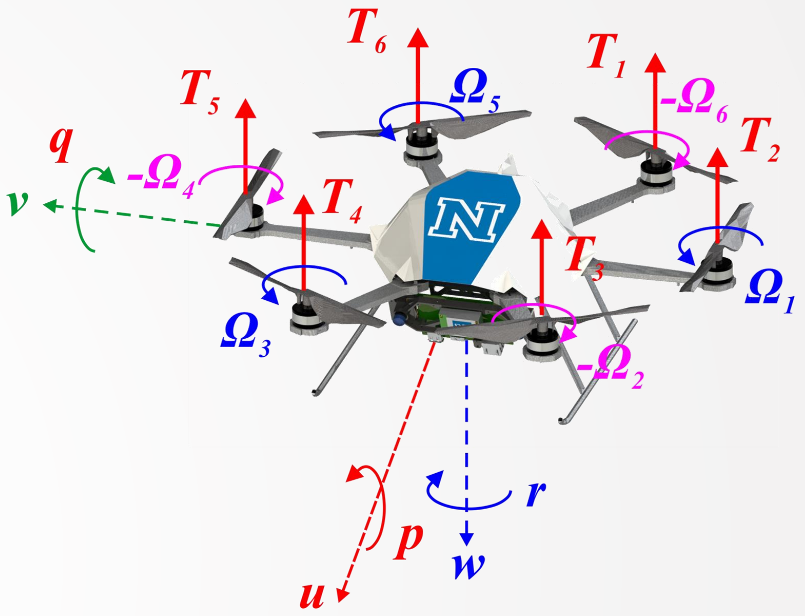
$$\frac{d\mathbf{V}_g}{dti} = \frac{d\mathbf{V}_g}{dt_b} + \omega_{b/i} \times \mathbf{V}_g \quad \Rightarrow \quad m(\frac{d\mathbf{V}_g}{dt_b} + \omega_{b/i} \times \mathbf{V}_g) = \mathbf{f}$$

- Which expressed in the body frame:

$$m(\frac{d\mathbf{V}_g^b}{dt_b} + \omega_{b/i}^b \times \mathbf{V}_g^b) = \mathbf{f}^b$$



MAV Dynamics



$$m\left(\frac{d\mathbf{V}_g^b}{dt_b} + \omega_{b/i}^b \times \mathbf{V}_g^b\right) = \mathbf{f}^b$$

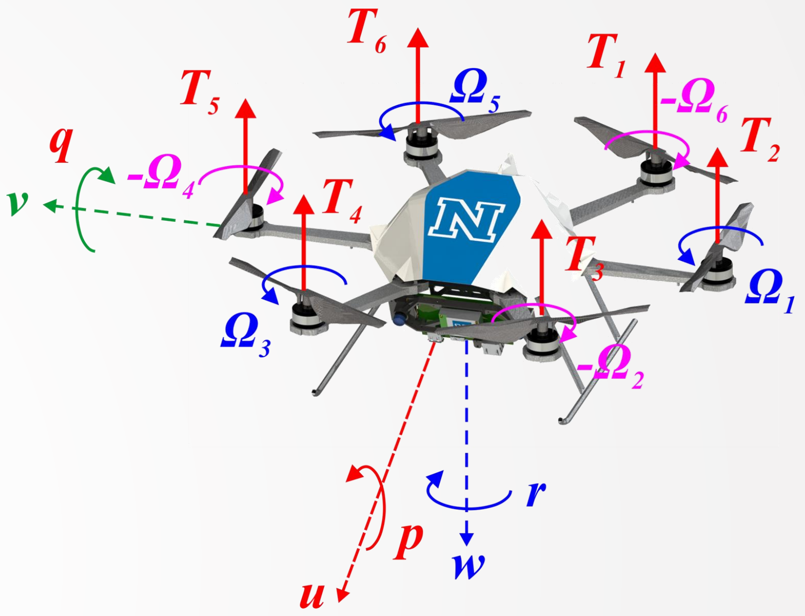
- Where

$$\mathbf{V}_g^b = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \omega_{b/i}^b = \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \mathbf{f}^b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

- Therefore:

$$\boxed{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} + \frac{1}{m}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}}$$

MAV Dynamics



- Recall Newton's 2nd Law:

$$\frac{d\mathbf{h}}{dt_i} = \mathbf{m}$$

  - ***h*** is the angular momentum vector
  - ***m*** is the summary of all external moments
  - Time derivative is taken wrt the interial frame
- Therefore:

$$\frac{d\mathbf{h}}{dt_i} = \frac{d\mathbf{h}}{dt_b} + \omega_{b/i} \times \mathbf{h} = \mathbf{m}$$

- Which expressed in the body frame:

$$\frac{d\mathbf{h}^b}{dt_b} + \omega_{b/i}^b \times \mathbf{h}^b = \mathbf{m}^b$$

MAV Dynamics

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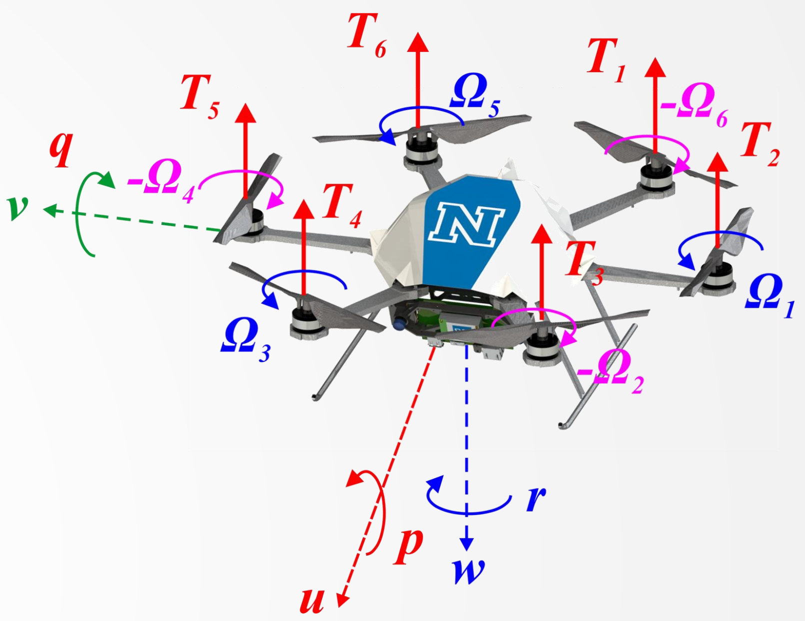


MAV Dynamics

- For a rigid body, the angular momentum is defined as the product of the inertia matrix and the angular velocity vector:

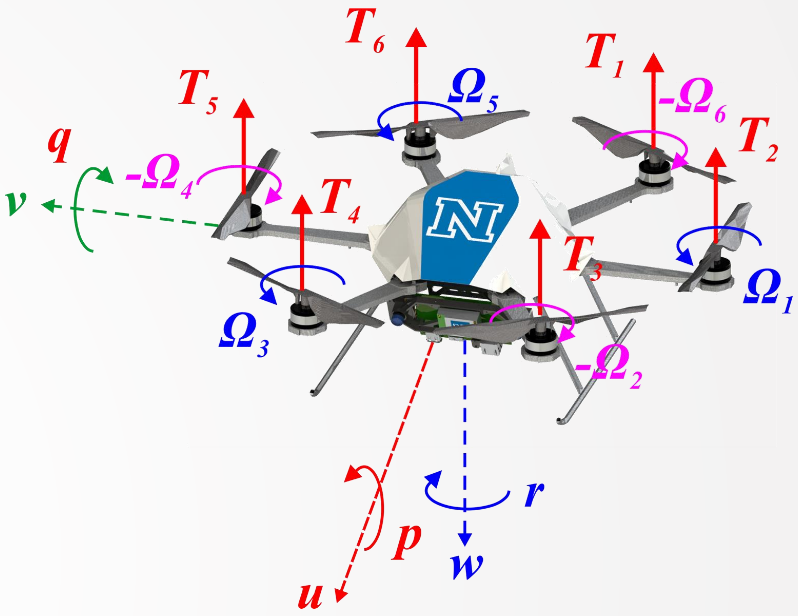
$$\mathbf{h}^b = \mathbf{J}\omega^b_{b/i}$$

- where

$$\mathbf{J} = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{xy} & J_y & -J_{yz} \\ -J_{xz} & -J_{yz} & J_z \end{bmatrix}$$

- But as the multirotor MAV is symmetric:

$$\mathbf{J} = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix}$$



MAV Dynamics

- Replacing in:

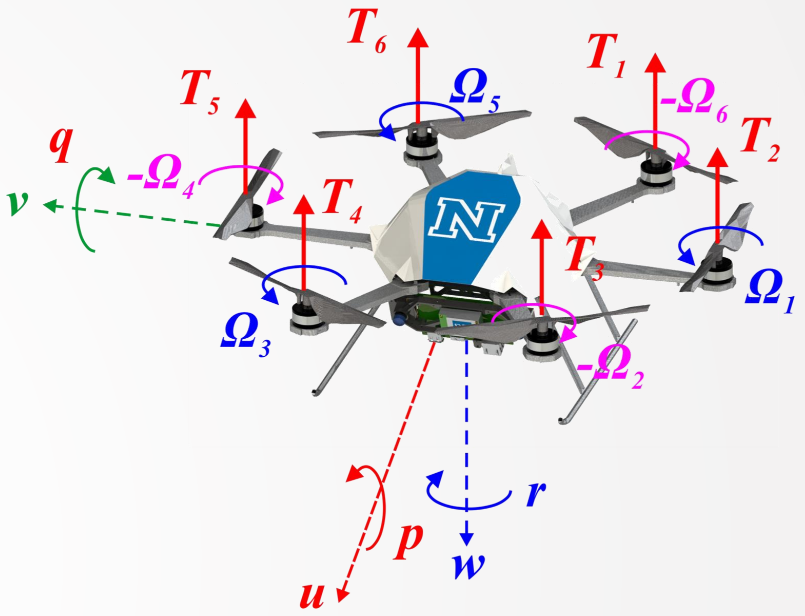
$$\frac{d\mathbf{h}^b}{dt_b} + \omega^b_{b/i} \times \mathbf{h}^b = \mathbf{m}^b$$

- Gives:

$$\mathbf{J}\frac{d\omega^b_{b/i}}{dt_b} + \omega^b_{b/i} \times (\mathbf{J}\omega^b_{b/i}) = \mathbf{m}^b \Rightarrow$$
$$\dot{\omega}^b_{b/i} = \mathbf{J}^{-1}[-\omega^b_{b/i} \times (\mathbf{J}\omega^b_{b/i}) + \mathbf{m}^b]$$

- where

$$\dot{\omega}^b_{b/i} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}$$



MAV Dynamics

- By setting the moments vector:

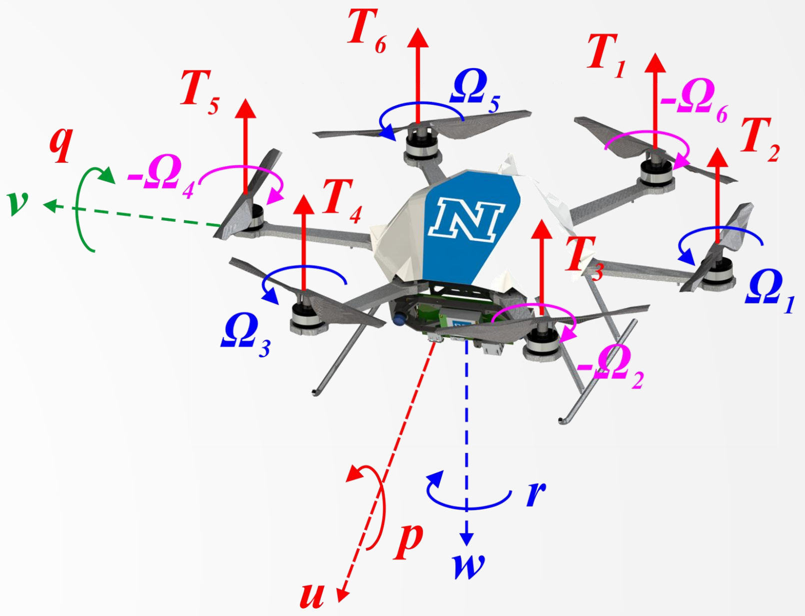
$$\mathbf{m}^b = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

- Then for the symmetric MAV, equation:

$$\dot{\omega}^b_{b/i} = \mathbf{J}^{-1}[-\omega^b_{b/i} \times (\mathbf{J}\omega^b_{b/i}) + \mathbf{m}^b]$$

- Becomes:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{J_y - J_z}{J_x} qr \\ \frac{J_z - J_x}{J_y} pr \\ \frac{J_x - J_y}{J_z} pq \end{bmatrix} + \begin{bmatrix} \frac{1}{J_x} M_x \\ \frac{1}{J_y} M_y \\ \frac{1}{J_z} M_z \end{bmatrix}$$



MAV Dynamics

- To append the forces and moments we need to combine their formulation with

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$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{bmatrix} = \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} + \frac{1}{m}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

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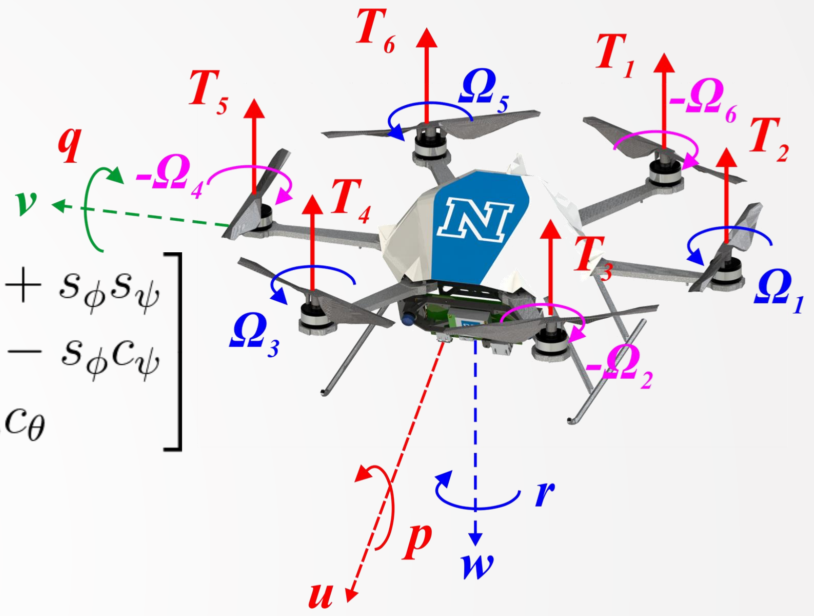
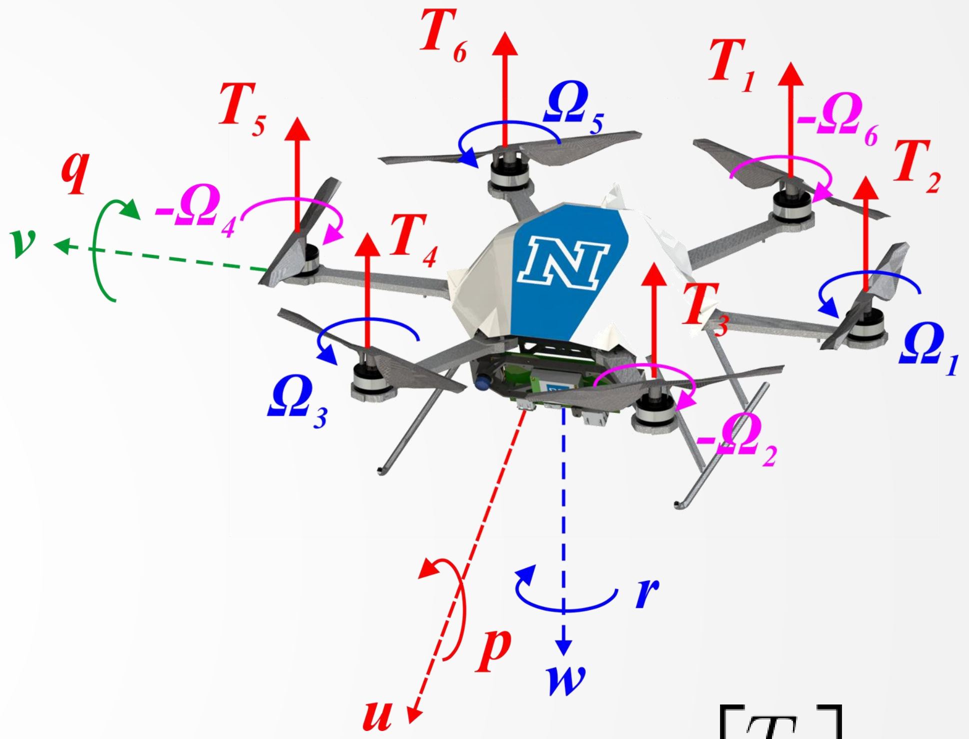


- ***Next step: append the MAV forces and moments***

MAV Dynamics

- MAV forces in the body frame:

$$\mathbf{f}_b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{6} T_i \end{bmatrix}$$



- Moments in the body frame:

$$\mathbf{m}_b = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} lc_{60} & l & lc_{60} & -lc_{60} & -l & -lc_{60} \\ -ls_{60} & 0 & ls_{60} & ls_{60} & 0 & -ls_{60} \\ -k_m & k_m & -k_m & k_m & -k_m & k_m \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

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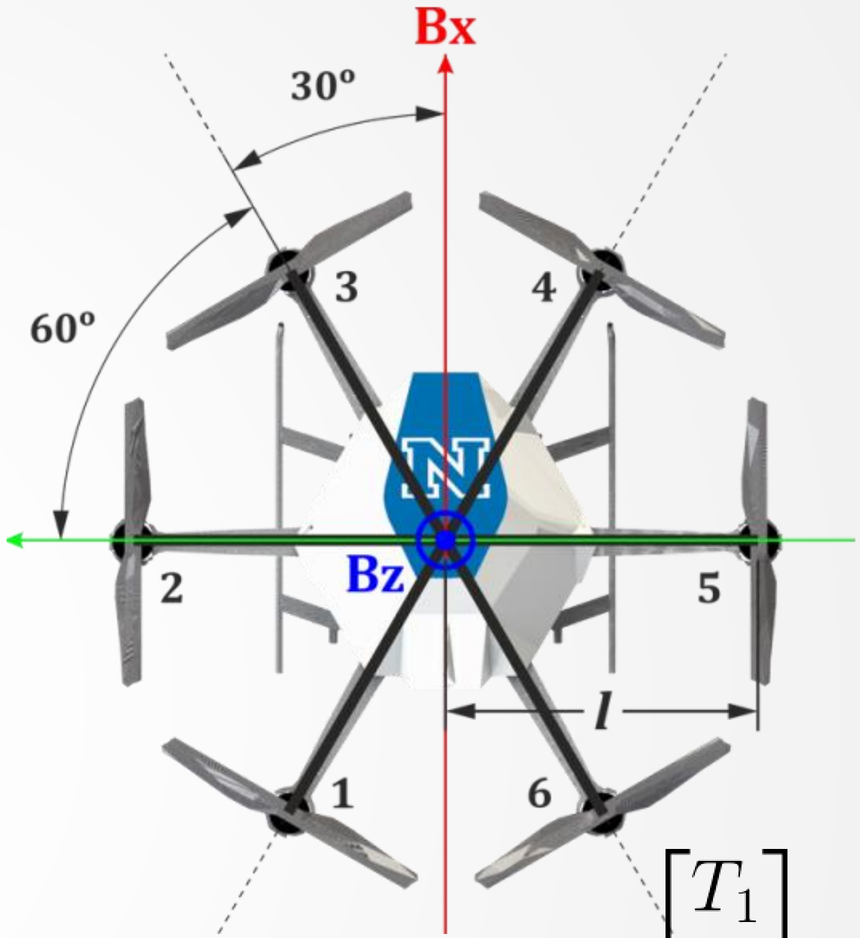

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Code Example

- MATLAB Quadrotor Simulator:
  - https://github.com/unr-arl/drones_demystified/tree/master/matlab/vehicle-dynamics
  - `Accurate dynamics simulator with further realistic features on sensing data and planning algorithms.`
  - `Create by: Ke Sun, University of Pennsylvania`
  - `run “quad_sim.m”`

- ROS/Gazebo Multirotor Simulator:
  - http://www.autonomousrobotslab.com/rotors-simulator2.html
  - Advanced aerial robots simulator, recreating real-life autonomous operation in terms of actuation, dynamics, control systems, sensor systems, localization algorithms, as well as path planning.
  - Very realistic – relying on Gazebo and physics engine.
  - roslaunch rotors_gazebo mav_hovering_example.launch mav_name:=firefly world_name:=basic

Find out more

- S. Leutenegger, C. Huerzeler, A.K. Stowers, K. Alexis, M. Achtelik, D. Lentink, P. Oh, and R. Siegwart. **"Flying Robots"**, *Handbook of Robotics* (upcoming new version – available upon request).
- Python? http://www.autonomousrobotslab.com/simulations-with-simpy.html
- **Help with Differential Equations?** https://www.khanacademy.org/math/differential-equations
- **Always check:** http://www.kostasalexis.com/literature-and-links1.html

Thank you!

Please ask your question!