Drones Demystified!

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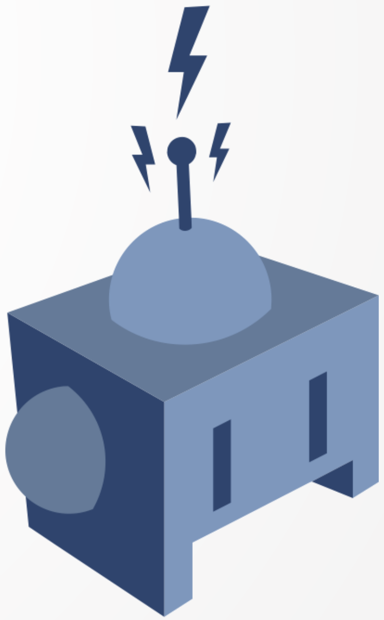
Topic: State Estimation

How do I estimate my position?

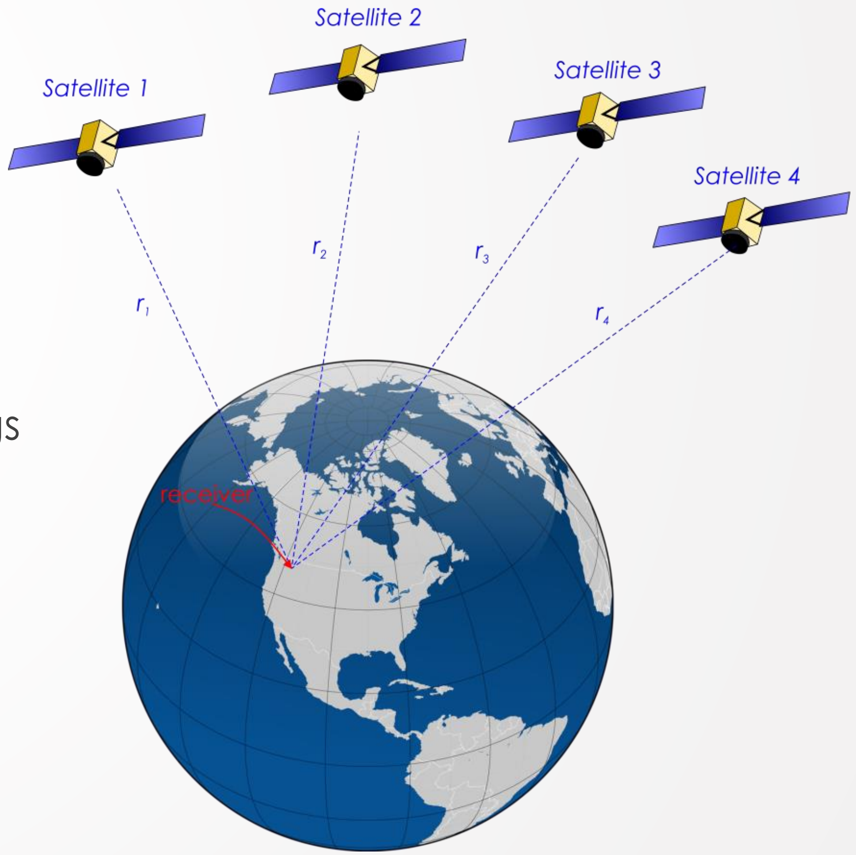


World state (or system state)

- *Belief state:*
  - **Our belief/estimate of the world state**
- *World state:*
  - **Real state of the robot in the real world**

State Estimation

- **What parts of the world state are (most) relevant for a flying robot?**
  - **Position**
  - **Velocity**
  - **Orientation**
  - **Attitude rate**
  - Obstacles
  - Map
  - Positions and intentions of other robots/human beings
  - …



State Estimation

- **Cannot observe world state directly – no sensor tells us where we really are but rather some measurements related to that (and noisy!)**
- **Need to estimate the world state**
  - **But How?**
    - Infer world state from sensor data
    - Infer world state from executed motions/actions

Sensor Model

- Robot perceives the environment through its sensors:

$$\mathbf{z} = h(\mathbf{x})$$

  - Where $\mathbf{z}$ is **the sensor reading**, $\mathbf{h}$ is the **world state**.

- **Goal:** Infer the state of the world from sensor readings.

$$\mathbf{x} = h^{-1}(\mathbf{z})$$

Motion Model

- Robot executes an action (or control) $\mathbf{u}$
  - e.g: move forward at 1m/s

- Update **belief state** according to the **motion model**:

$$\mathbf{x}' = g(\mathbf{x}, \mathbf{u})$$

  - Where $\mathbf{x}'$ is the current state and $\mathbf{x}$ is the previous state.

Probabilistic Robotics

- Sensor observations are noisy, partial, potentially missing.
- All models are partially wrong and incomplete.
- Usually we have prior knowledge.

Probabilistic Robotics

- Probabilistic sensor models: $\mathbf{p}(\mathbf{z}|\mathbf{x})$
- Probabilistic motion models: $\mathbf{p}(\mathbf{x}'|\mathbf{x}, \mathbf{u})$
- Fuse data between multiple sensors (multi-modal):

$$\mathbf{p}(\mathbf{x}|\mathbf{z}_{GPS}, \mathbf{z}_{BARO}, \mathbf{z}_{IMU})$$

- Fuse data over time (filtering):

$$\mathbf{p}(\mathbf{x}|\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_t)$$

$$\mathbf{p}(\mathbf{x}|\mathbf{z}_1, \mathbf{u}_1, \mathbf{z}_2, \mathbf{u}_2, \ldots, \mathbf{z}_t, \mathbf{u}_5)$$

Drones Demystified!

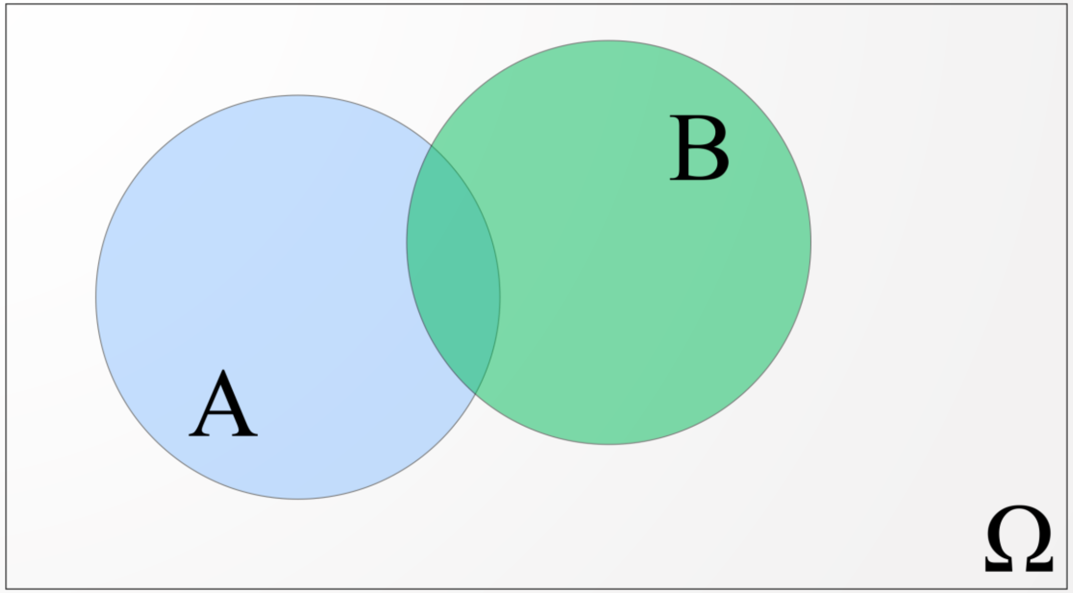
Topic: State Estimation – Recap on Probabilities

Probability theory

- **Random experiment** that can produce a number of outcomes, e.g. a rolling dice.
- Sample space, e.g.: $\{1,2,3,4,5,6\}$
- Event A is subset of outcomes, e.g. $\{1,3,5\}$
- Probability $P(A)$, e.g. $P(A)=0.5$

Axioms of Probability theory

- $0 \leq P(A) \leq 1$
- $P(\Omega) = 1,\ P(\emptyset) = 0$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Discrete Random Variables

- X denotes a random variable
- X can take on a countable number of values in $\{x_1, x_2, \ldots, x_n\}$
- $P(X=x_i)$ is the probability that the random variable X takes on value x_i
- $P(.)$ is called the probability mass function

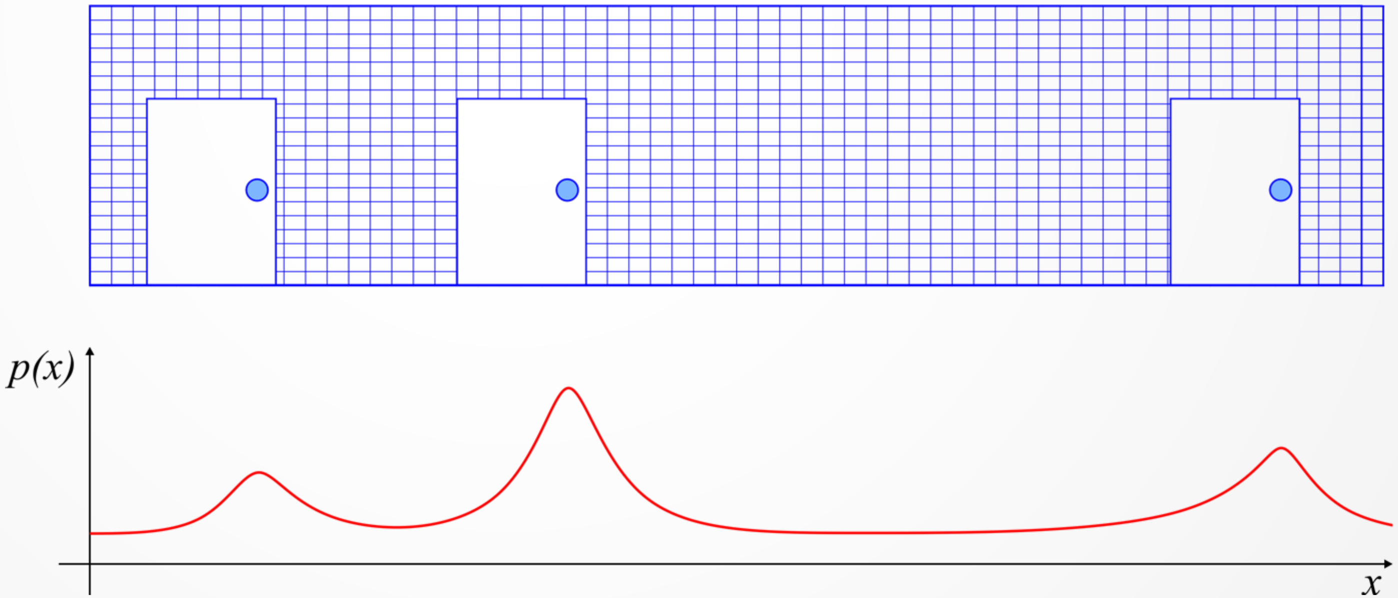
- Example: P(Room)=<0.6,0.3,0.06,0.03>, Room one of the office, corridor, lab, kitchen

Continuous Random Variables

- X takes on continuous values.
- P(X=x) or P(x) is called the **probability density function (PDF)**.

Thrun, Burgard, Fox, "Probabilistic Robotics", MIT Press, 2005

- Example:



Proper Distributions Sum To One

- Discrete Case $$\sum_x P(x) = 1$$
- Continuous Case $$\int p(x)dx = 1$$

Joint and Conditional Probabilities

- $p(X = x, \text{ and } Y = y) = P(x, y)$
- If X and Y are **independent** then:

$$P(x, y) = P(x)P(y)$$

- Is the probability of **x given y**

$$P(x|y)P(y) = P(x, y)$$

- If X and Y are independent then:

$$P(x|y) = P(x)$$

Conditional Independence

- Definition of conditional independence:

$$P(x, y|z) = P(x|z)P(y|z)$$

- Equivalent to:

$$P(x|z) = P(x|y, z)$$
$$P(y|z) = P(y|x, z)$$

- Note: this does not necessarily mean that:

$$P(x, y) = P(x)P(y)$$

Marginalization

- **Discrete case:** $$P(x) = \sum_y P(x, y)$$
- **Continuous case:** $$p(x) = \int p(x, y)dy$$

Marginalization example

P(X,Y)	x1	x1	x1	x1	P(Y) ↓
y1	1/8	1/16	1/32	1/32	1/4
y1	1/16	1/8	1/32	1/32	1/4
y1	1/16	1/16	1/16	1/16	1/4
y1	1/4	0	0	0	1/4
P(X) →	1/2	1/4	1/8	1/8	1

Expected value of a Random Variable

- **Discrete case:** $$E[X] = \sum_i x_i P(x_i)$$

- **Continuous case:** $$E[X] = \int x P(X = x) dx$$

- The expected value is the weighted average of all values a random variable can take on.
- Expectation is a linear operator:

$$E[aX + b] = aE[X] + b$$

Covariance of a Random Variable

- Measures the **square expected deviation from the mean**:

$$Cov[X] = E[X - E[X]]^2 = E[X^2] - E[X]^2$$

Estimation from Data

- Observations: $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n \in \mathcal{R}^d$
- Sample Mean: $\mu = \frac{1}{n}\sum_i \mathbf{x}_i$
- Sample Covariance:

$$\Sigma = \frac{1}{n-1}\sum_i (\mathbf{x}_i - \mu)(\mathbf{x}_i - \mu)$$

Find out more

- http://www.autonomousrobotslab.com/the-kalman-filter.html
- http://aerostudents.com/files/probabilityAndStatistics/probabilityTheoryFullVersion.pdf
- http://www.cs.unc.edu/~welch/kalman/
- http://home.wlu.edu/~levys/kalman_tutorial/
- https://github.com/rlabbe/Kalman-and-Bayesian-Filters-in-Python
- http://www.autonomousrobotslab.com/literature-and-links.html

Thank you!

Please ask your question!