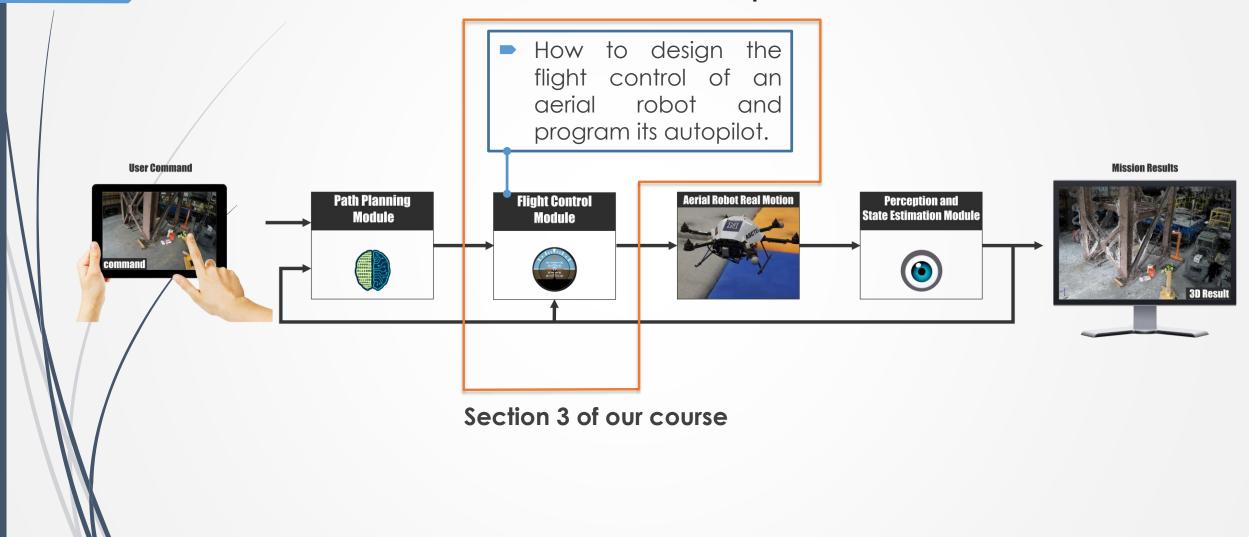
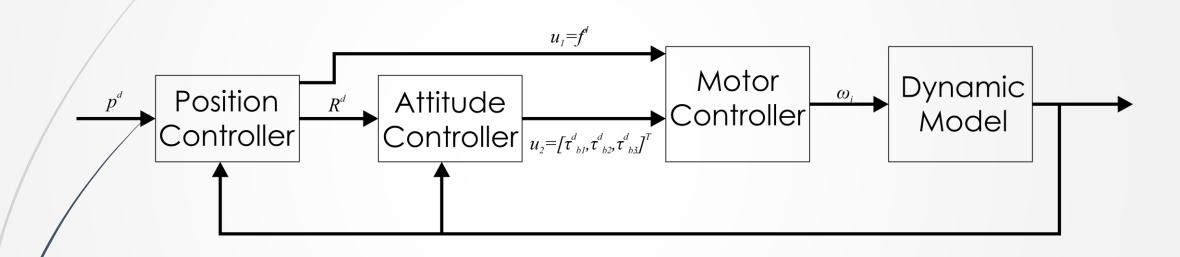


The Aerial Robot Loop



Control System Block Diagram



Simplified loop

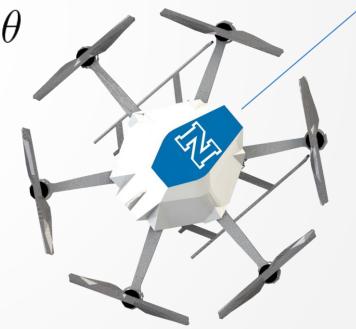


- Assume a single-axis case.
 - The system has to coordinate its pitching motion and thrust to move to the desired point ahead of its axis.
 - Roll is considered to be zero, yaw is considered to be constant. No initial velocity. No motion is expressed in any other axis.
 - A system of only two degrees of freedom.

Simplified linear dynamics

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_y \end{bmatrix} M_y$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -g \end{bmatrix} \theta$$



Explanation of translation model

$$ma = F_x \Rightarrow ma = T_{TOT}\cos(-\theta) \Rightarrow ma \approx -\theta T_{TOT}$$

$$ma = -\theta mg$$

$$ma = -\theta mg$$

s.t. $T_{TOT} = mg$
 $a = \ddot{x}$

$$a = \ddot{x}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -g \end{bmatrix} \theta$$

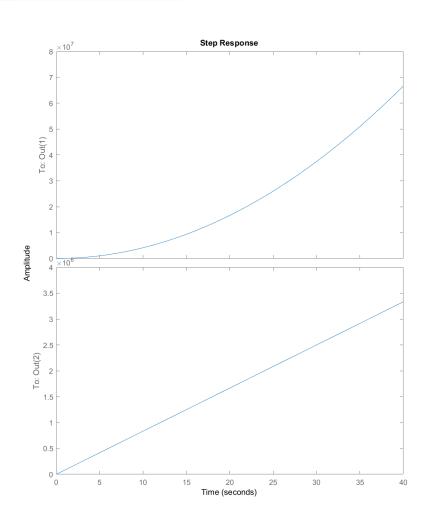
Simplified linear dynamics

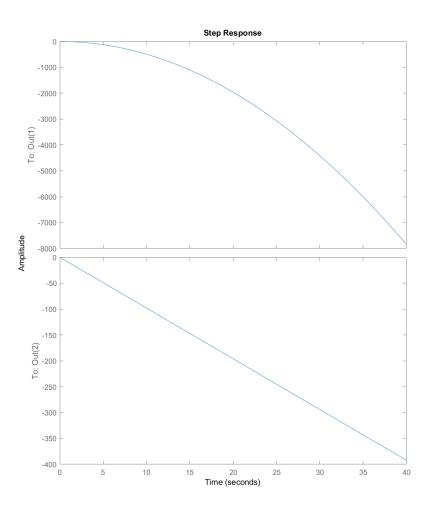
$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_y \end{bmatrix} M_y$$

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How does this system behave?







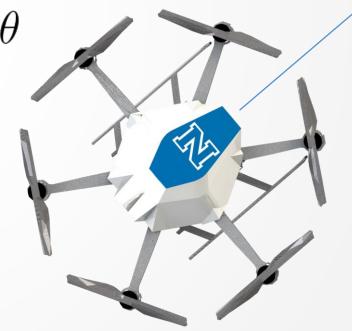
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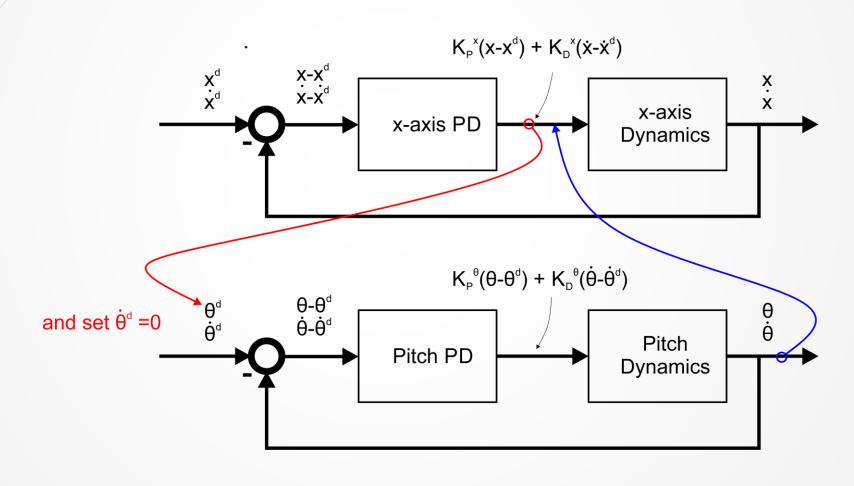
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How to control this system?

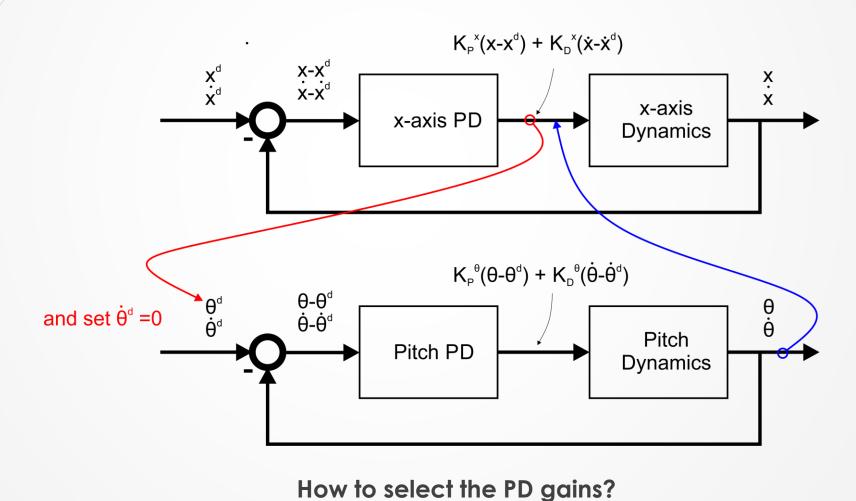
Most common way: use of PD controllers



Decoupled Control Structure

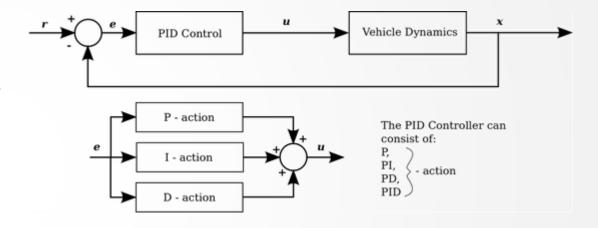


Decoupled Control Structure



A mini introduction to PID Control

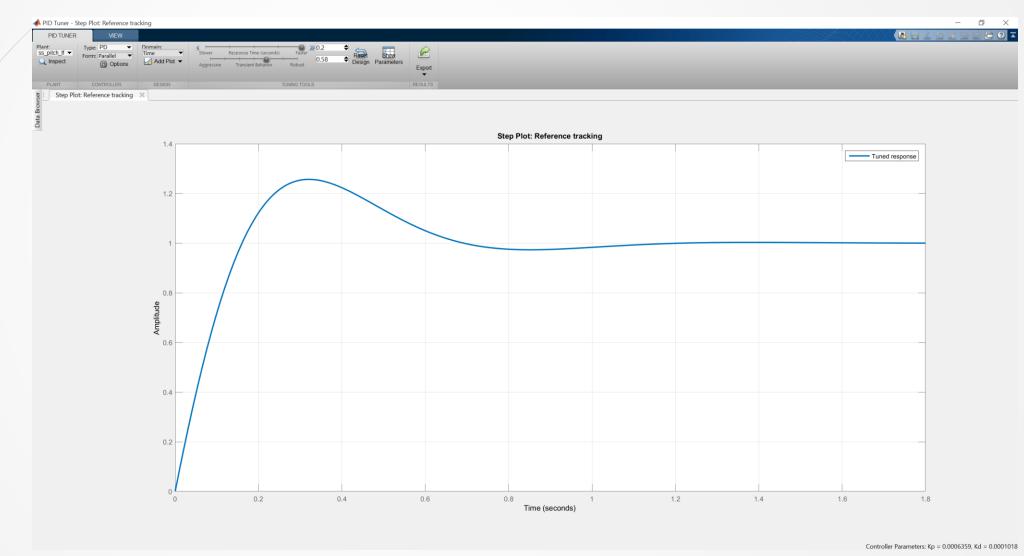
- PID = Proportional-Integral-Derivative feedback control
- It corresponds to one of the most commonly used controllers used in industry.
- It's success is based on its capacity to efficiently and robustly control a variety of processes and dynamic systems, while having an extremely simple structure and intuitive tuning procedures.
- Not comparable in performance with modern control strategies, but still the most common starting point



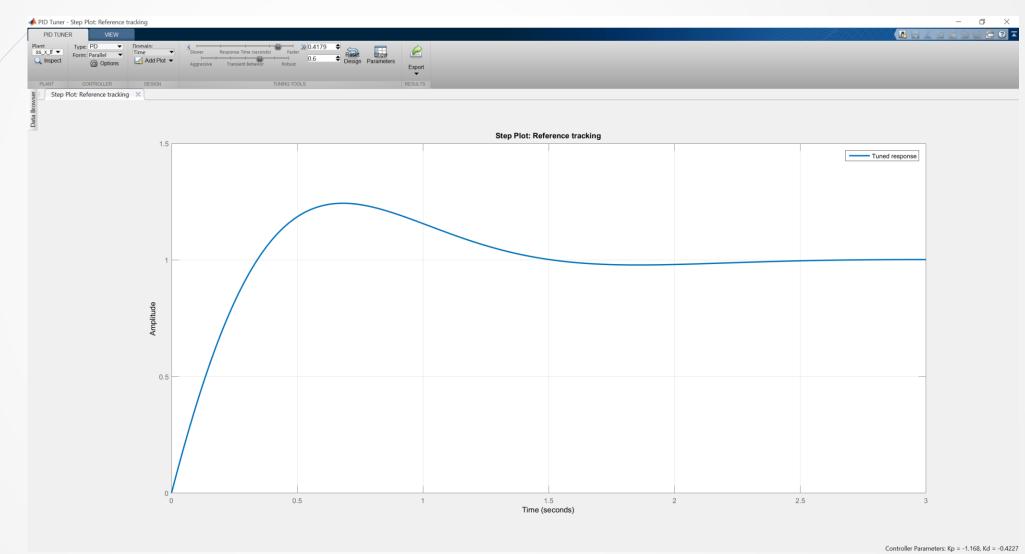
How to select the PD gains?

```
%% Simple Modeling and Control study
clear;
                                                                                              MATLAB Implementation
J y = 1.2e-5;
q = 9.806; mass = 1.2;
% Pitch Linear Model
A p = [0 1; 0 0]; B p = [0; 1/J y];
C p = eye(2); D p = zeros(2,1);
ss pitch = ss(A p,B p,C p,D p);
% x Linear Model
A x = [0 1; 0 0]; B x = [0; -q];
C x = eye(2); D x = zeros(2,1);
ss x = ss(A x, B x, C x, D x);
% Observe the Step responses of the system
subplot(1,2,1); step(ss pitch);
subplot(1,2,2); step(ss x);
%% Design the PD Controller for Pitch
ss pitch tf = tf(ss pitch); ss pitch tf = ss pitch tf(1);
pidTuner(ss pitch tf,'PD')
%% Design the PD Controller for X translational dynamics
ss x tf = tf(ss x); ss x tf = ss x tf(1);
pidTuner(ss x tf,'PD')
%% Verification
close all;
K_P_pitch = 25.8e-5; K_D_pitch = 9.82e-5;
PD PITCH GAINS = [K P pitch 0; 0 K D pitch];
ss pitch cl = feedback(PD PITCH GAINS*ss pitch,[1 1]);
K P X = -1.17; K D X = -0.823;
PD X GAINS = [K P x 0; 0 K D x];
ss_x_cl = feedback(PD_X GAINS*ss_x,[1 1]);
subplot(1,2,1); step(ss_pitch_cl);
subplot(1,2,2); step(ss x cl);
```

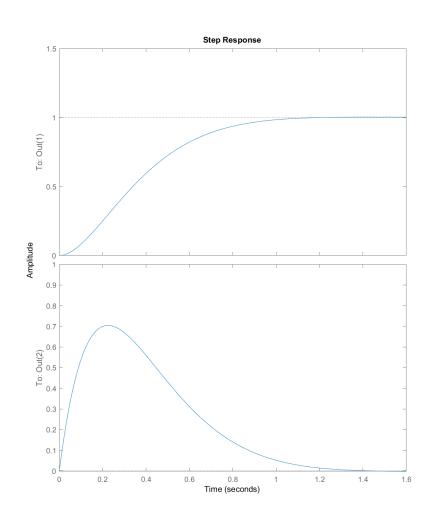
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subplot(1,2,1); step(ss pitch cl);
subplot(1,2,2); step(ss x cl);
```

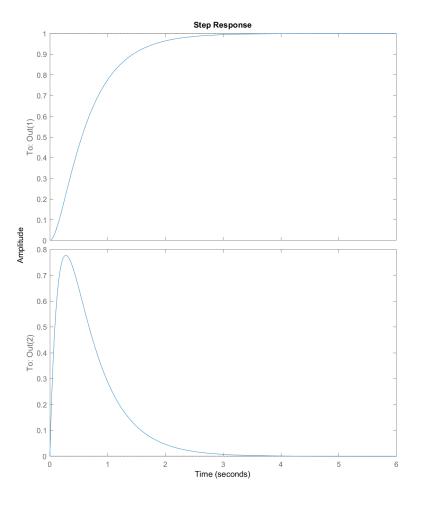


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ss_x_t = tf(ss_x); ss_x_t = ss_x_t (1);
pidTuner(ss x tf, 'PD')
%% Verification
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ss x cl = feedback(PD X GAINS*ss x,[1 1]);
subplot(1,2,1); step(ss pitch cl);
subplot(1,2,2); step(ss x cl);
```





Real-life Limitations

- The control margins of the aerial vehicle have limits and therefore the PID controller has to be designed account for these constraints.
- The integral term needs special caution due to the often critically stable or unstable characteristics expressed by unmanned aircraft.
- With the exception of hover/or trimmed-flight, an aerial vehicle is a nonlinear system. As the PID is controller, it naturally cannot maintain an equally good behavior for the full flight envelope of the system. A variety of techniques such as Gain scheduling are employed to deal with this fact.

Code Examples and Tasks



- <u>https://github.com/unr-arl/drones_demystified/tree/master/matlab/control-systems/gain-scheduled-three-loop-aircraft-autopilot</u>
- <u>https://github.com/unr-arl/drones_demystified/tree/master/matlab/control-systems/lqr</u>
- <u>https://github.com/unr-arl/drones_demystified/tree/master/matlab/control-systems/pid-cruise-control</u>
- <u>https://github.com/unr-arl/drones_demystified/tree/master/matlab/control-systems/pid</u>

Find out more

- http://www.autonomousrobotslab.com/pid-control.html
- <u>http://www.autonomousrobotslab.com/lqr-control.html</u>
- http://www.autonomousrobotslab.com/linear-model-predictive-control.html
- <u>http://ctms.engin.umich.edu/CTMS/index.php?example=InvertedPendulum</u> <u>§ion=ControlStateSpace</u>

http://www.autonomousrobotslab.com/literature-and-links1.html

