Autonomous Mobile Robot Design

Topic: Wheeled Robot Kinematics

Dr. Kostas Alexis (CSE)

Goal of this lecture

- The goal of this lecture is to derive the kinematic equations for a wheeled robot.
- We will focus primarily on differential drive systems.

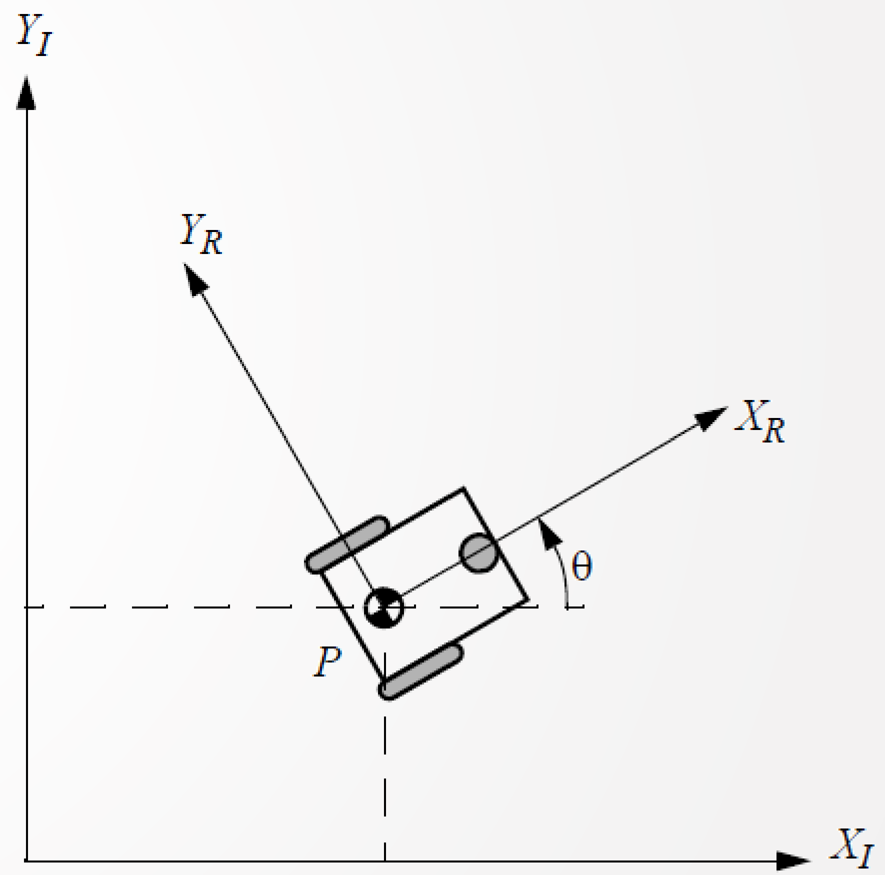
Representing Robot Pose

- The pose of a simplified robot in the 2D configuration space is defined by its x-y coordinates and the heading angle θ.

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- The heading angle of the robot affects its dynamic trajectory in the x-y space. Let us defined the rotation matrix:

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



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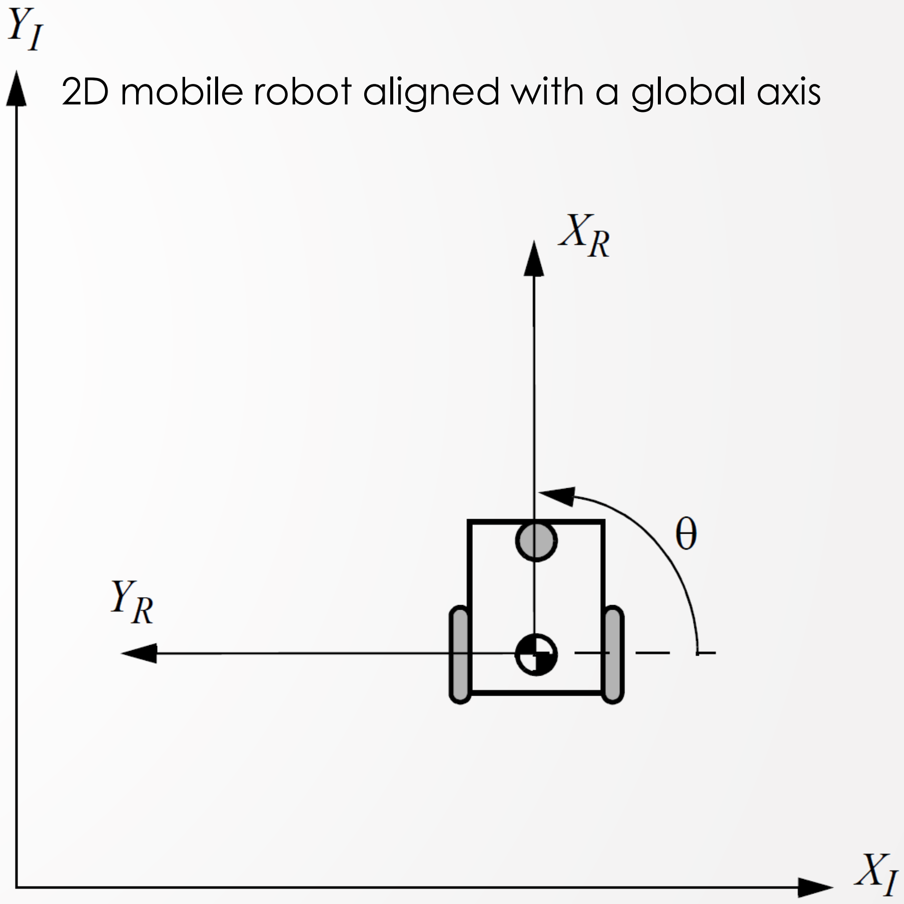


Representing Robot Pose

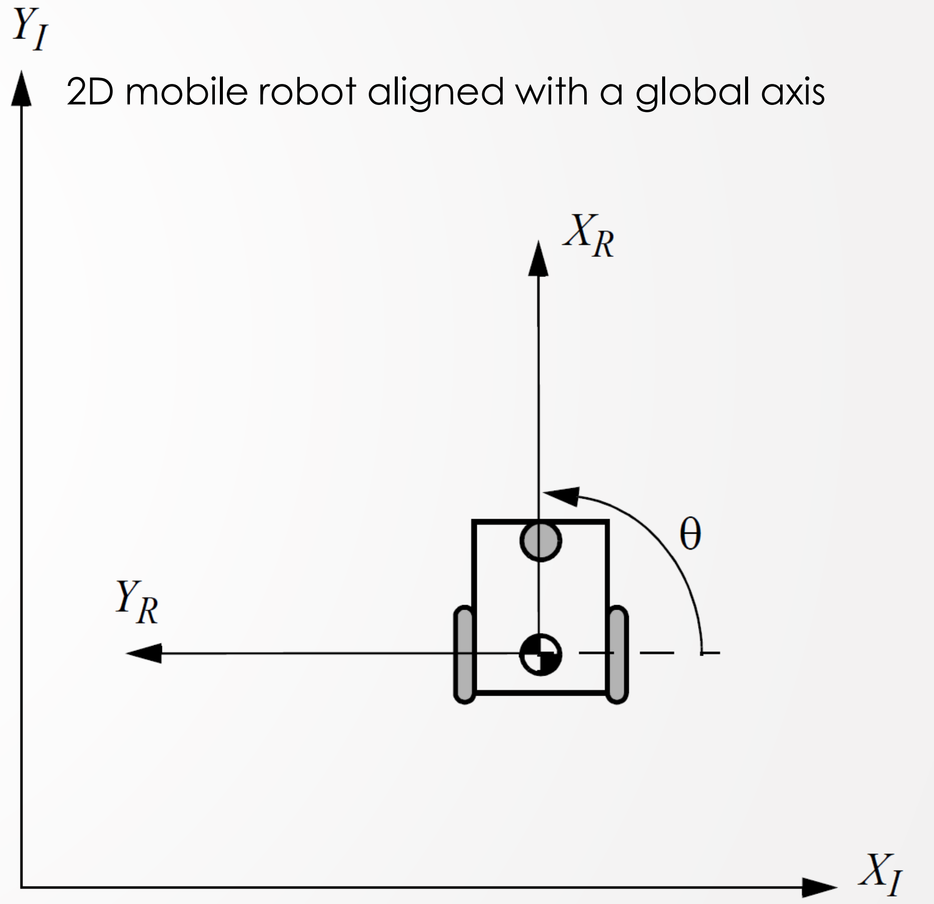
- The rotation matrix $R(\theta)$ can be used to map the motion in the global reference frame (X_I, Y_I) to motion expressed in the local reference frame (X_R, Y_R).

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$

- Example for a heading angle of 90 degrees

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I$$

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$



Forward Kinematics Model

- In the simplest cases, the equation

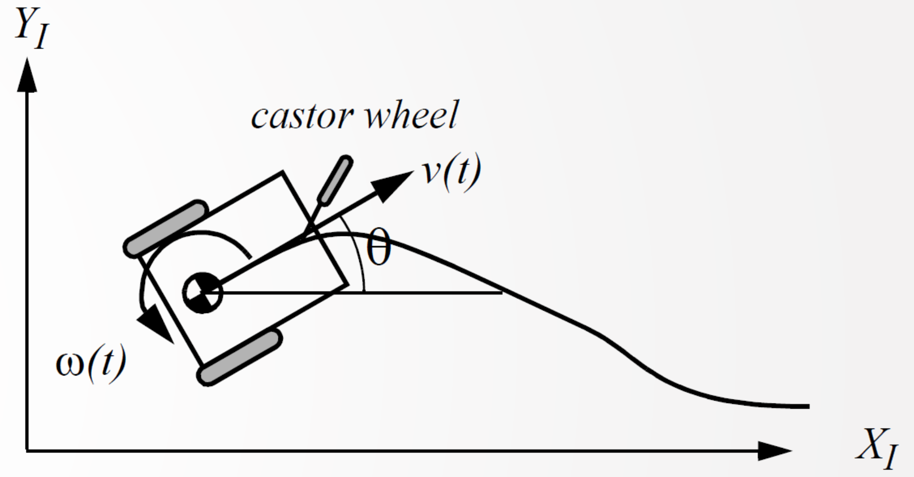
$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$

  is sufficient to describe the forward kinematics of a 2D mobile robot

- More generally:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$

- where φ_1, φ_2 the wheel angles (with corresponding turning rates ω_1, ω_2, wheel radius r and distance $2l$ between the two wheels.



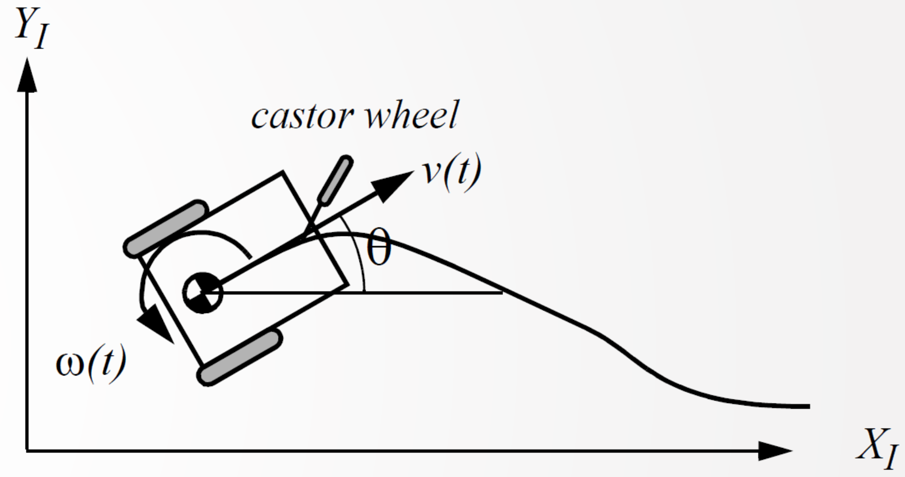
Forward Kinematics Model

- Then:

$$\omega_1 = \frac{r\dot{\varphi}_1}{2l} \quad \omega_2 = \frac{-r\dot{\varphi}_2}{2l}$$

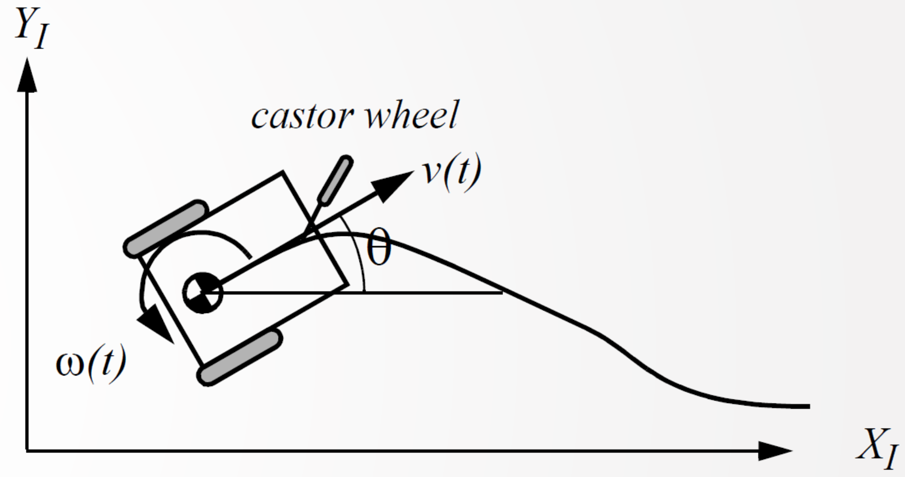
- Therefore:

$$\dot{\xi}_I = R(\theta)^{-1}\begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$



Wheel Kinematic Constraints

- Often wheels have constraints in their motion.
- In our analysis, we still assume certain simplifications, e.g. a) the plane of the wheel always remains vertical, b) there is –in all cases- a single point of contact between the wheel and the ground plane, c) there is no sliding at this single point of contact.

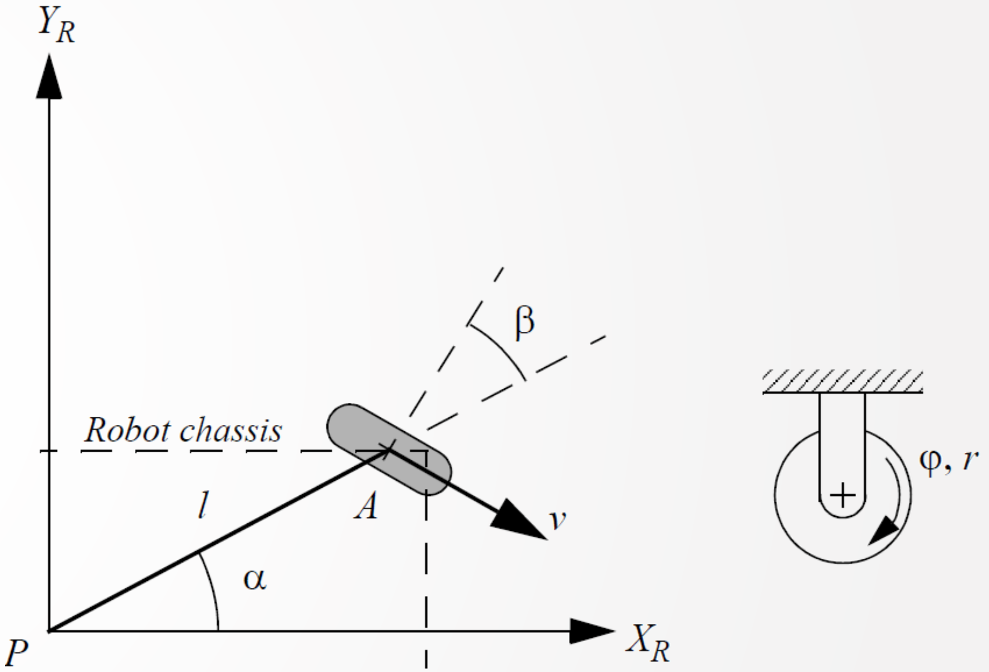


Fixed Standard Wheel

- The fixed standard wheel has no vertical axis of rotation for steering.
- Its angle to chassis is fixed and it is limited to motion back and forth along the wheel plane and rotation around its contact point with the ground plane.
- Expressing point A at polar coordinates:

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

  - The rolling constraint for this wheel enforces that all motion along the direction of the wheel plane must be accompanied by the appropriate amount of wheel spin so that there is pure rolling at the contact point.



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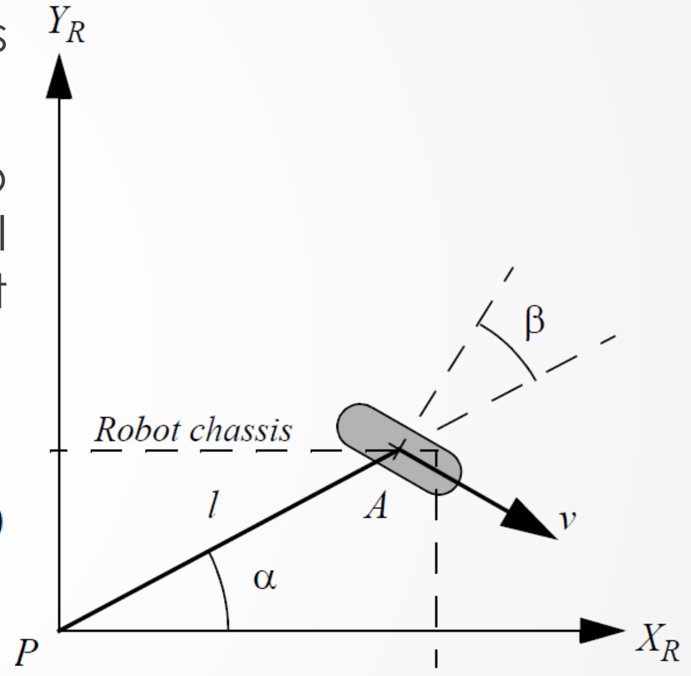
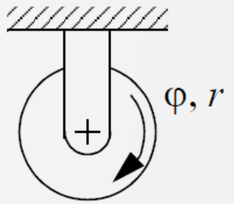
$$\left[\underbrace{\sin(\alpha+\beta)}_{\text{Total motion along the wheel plane}} \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$





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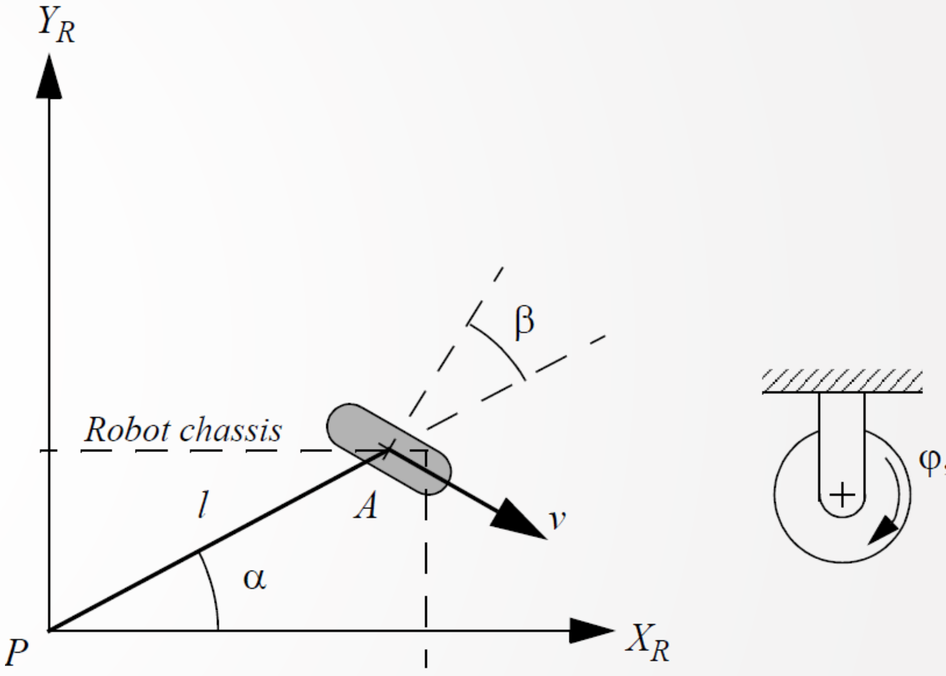
- Expressing point A at polar coordinates:

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

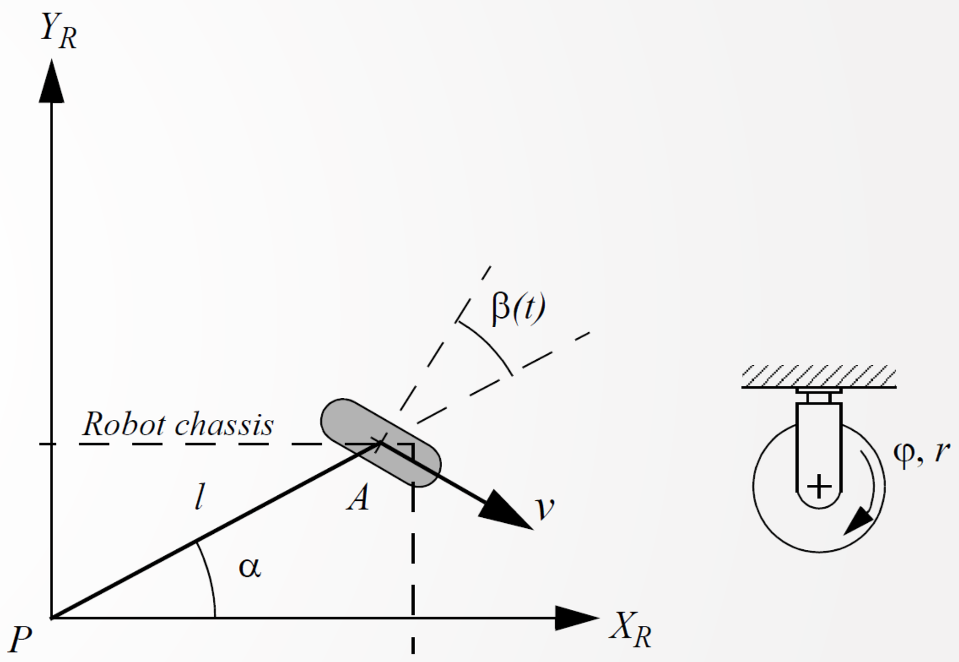
- Sliding constraint for this wheel enforces that the component of the wheel's motion orthogonal to the wheel plane must be zero:

$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$



Steered Standard Wheel

- The steered standard wheel differs from the fixed standard wheel only in that there is an additional degree of freedom: the wheel may rotate around a vertical axis passing through the center of the wheel and the ground contact point.
- The equations of position for the wheel are similar with the exception that the orientation of the wheel to the robot chassis is no longer a constant β but a function of time $\beta(t)$.



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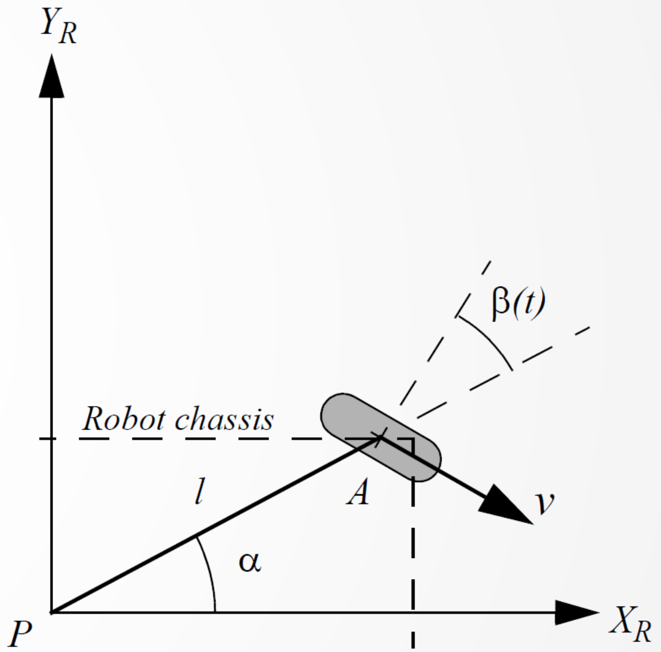
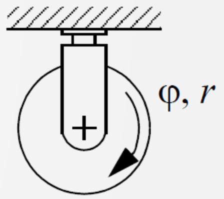
- The equations of position for the wheel are similar with the exception that the orientation of the wheel to the robot chassis is no longer a constant β but a function of time $\beta(t)$.
- Rolling and Sliding Constraints:

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

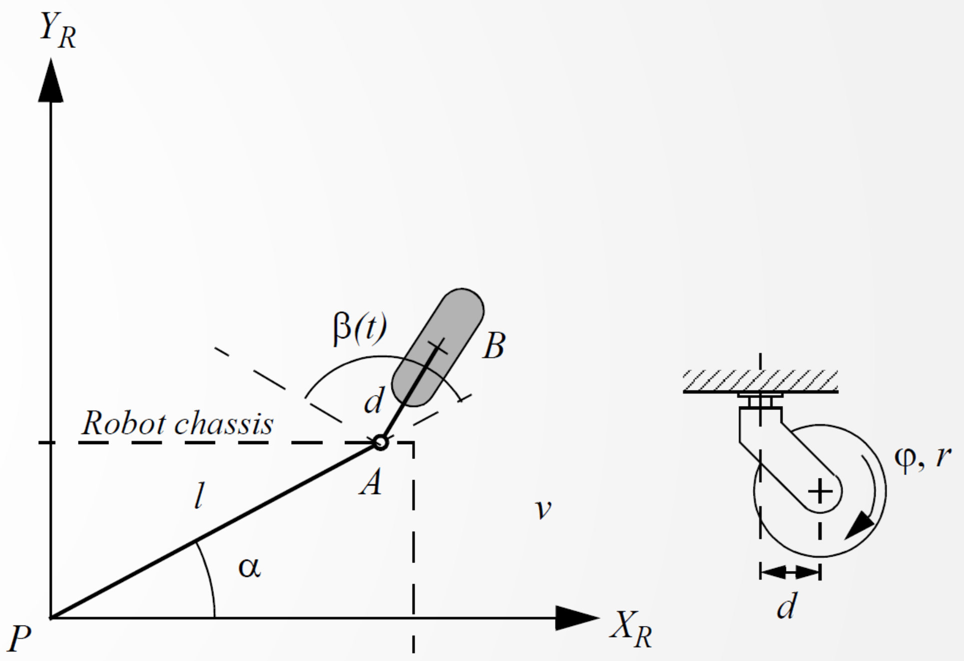
$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta\end{bmatrix} R(\theta)\dot{\xi}_I = 0 .$$





Castor Wheel

- Castor wheels are able to steer around a vertical axis but unlike steered standard wheel, the vertical axis of rotation in a castor wheel does not pass through the ground contact point.
- The wheel contact point is not at position B, which is connected by a rigid rod AB of fixed length d to point A, fixes the location of the vertical axis about which B steers, and this point A has a position specified in the robot's reference frame.



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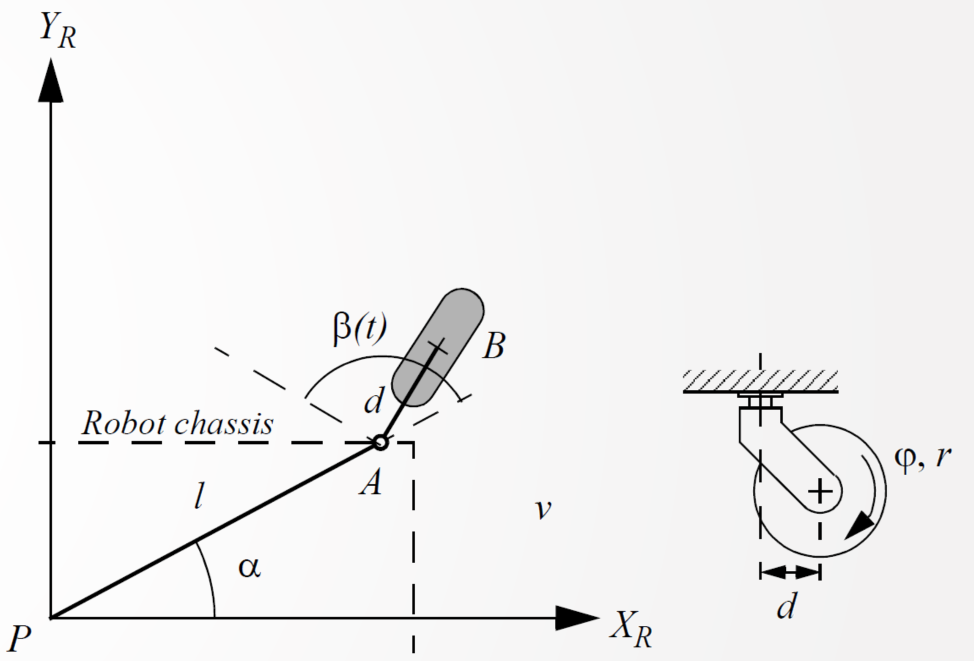
- Rolling Constraint:

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

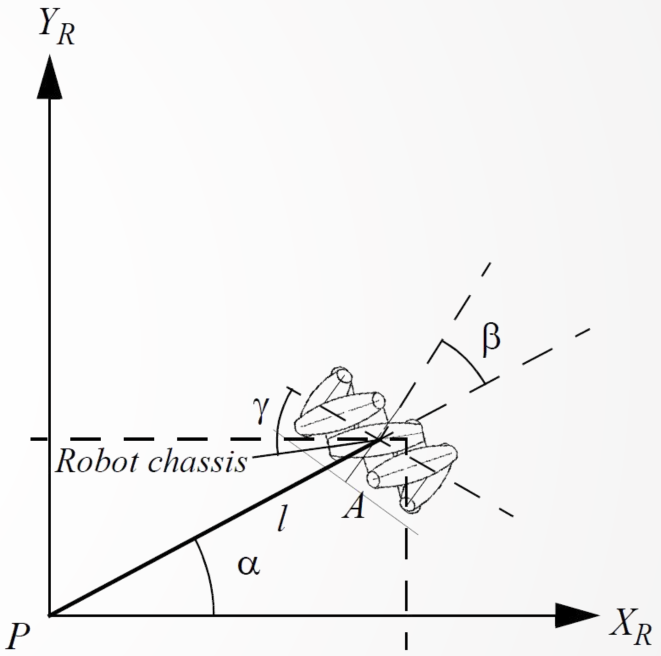
- Sliding Constraint:

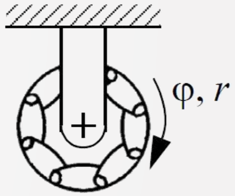
$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & d + l\sin\beta\end{bmatrix} R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$



Swedish Wheel

- Swedish wheels have no vertical axis of rotation, yet they are able to move omnidirectionally like the castor wheel.
- This is possible by adding a degree of freedom to the fixed standard wheel. Swedish wheels consist of a fixed standard wheel with rollers attached to the wheel perimeter with axes that are antiparallel to the main axis of the fixed wheel component.





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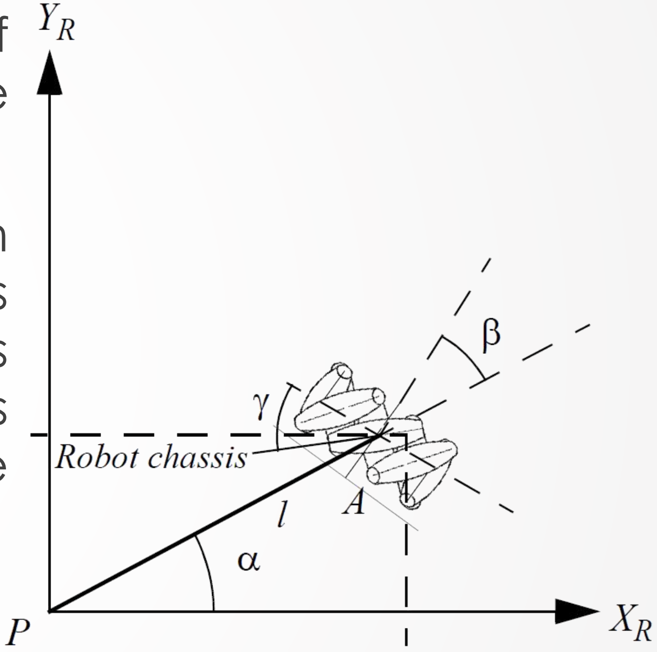
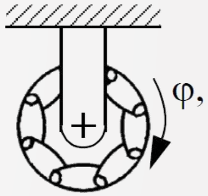
- Motion Constraint:

$$\left[\sin(\alpha+\beta+\gamma)\ \ -\cos(\alpha+\beta+\gamma)\ \ (-l)\cos(\beta+\gamma)\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}\cos\gamma = 0$$

$$\left[\cos(\alpha+\beta+\gamma)\ \ \sin(\alpha+\beta+\gamma)\ \ l\sin(\beta+\gamma)\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

- Angle γ is used such that the effective direction along which the rolling constraint holds is along this zero component and not along the wheel plane



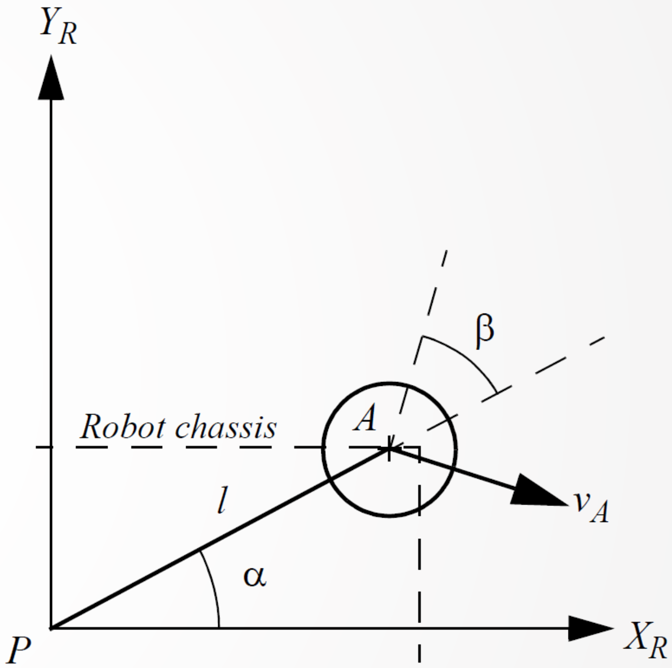


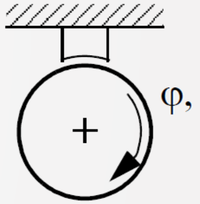
Spherical Wheel

- The spherical (or ball) wheel places no constraints on motion. It has no principal axis of rotation.
  - No appropriate rolling and sliding constraints.
- Motion equation of spherical wheel:

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$





Robot Kinematic Constraints

- Given a robot with M wheels we can use the knowledge of how each wheel imposes zero or more constraints on robot motion to form the overall robot kinematic constraints.
- We have organized the wheels in 5 categories (fixed, steerable, castor, Swedish and spherical) but the last three impose NO constraint on the robot chassis. Therefore, we only account for the fixed (f) and steerable (s) wheels.
  - $N = N_f + N_s$,N_f = number of fixed wheels, N_s = number of steered wheels with angles φ_f, φ_s
  - Then let:

$$\varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}$$

Robot Kinematic Constraints

- Robot Rolling Constraints:

$$J_1(\beta_s)R(\theta)\dot{\xi}_I - J_2\dot{\varphi} = 0$$

  - J_2 = diagonal matrix NxN whose entries are the radii r of al standard wheels
  - $J_1(\beta_s)$ = matrix with projections for all wheels to their motions along their individual wheel planes:

$$J_1(\beta_s) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}$$

Robot Kinematic Constraints

- Robot Sliding Constraints:

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

  - C_{1f} , C_{1s} = diagonal matrices (Nfx3, Nsx3) whose diagonal terms are the three terms in the equations:

$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right]R(\theta)\dot{\xi}_I = 0\,.$$

$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right]R(\theta)\dot{\xi}_I = 0$$

for all fixed and steering wheels correspondingly.

Robot Kinematic Constraints

- Robot Kinematic Constraints combined form:

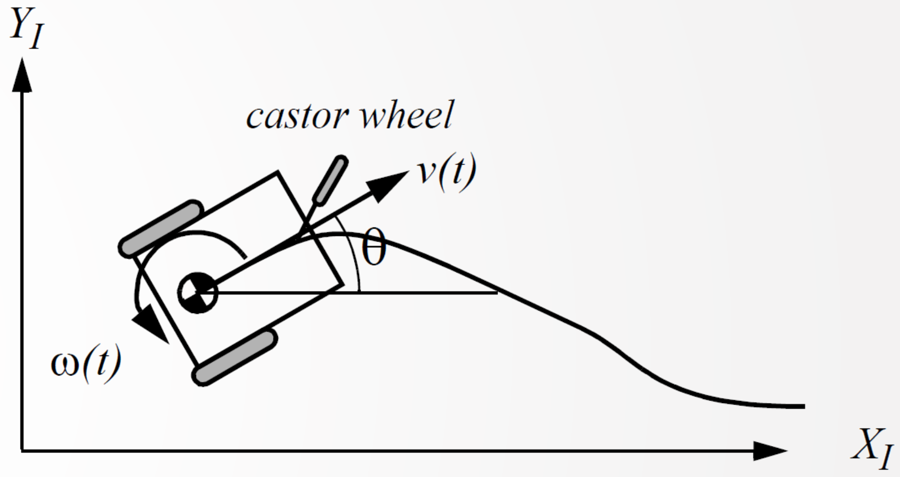
$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2\varphi \\ 0 \end{bmatrix}$$

Example: Differential Drive Robot

- Robot Kinematic Constraints combined form:

$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2\varphi \\ 0 \end{bmatrix}$$

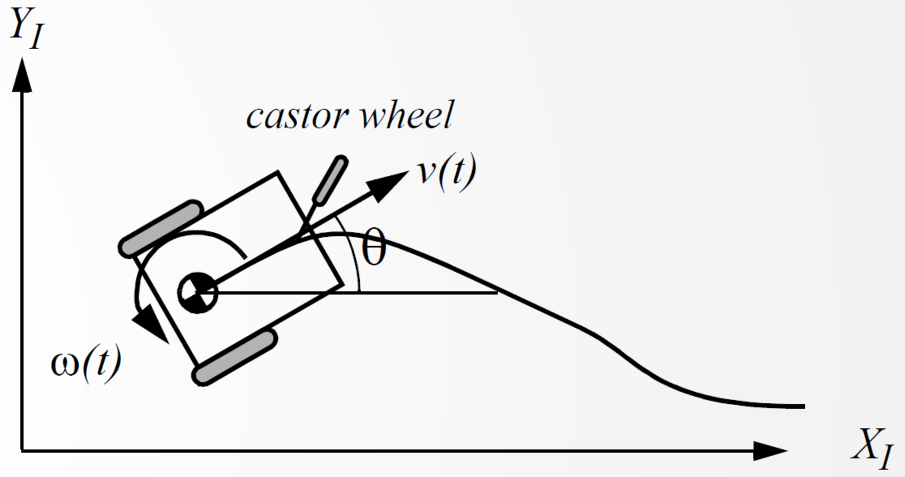
- The castor in unpowered and is free to move in any direction.
- The two remaining wheels are not steerable (so we have the fixed wheel formulas for each of them).
  - Supposing that the robot's local reference frame is aligned such that the robot moves forward along $+X_R$ then:
    - Right Wheel: $a = -\pi/2, \beta = \pi$
    - Left Wheel: $a = \pi/2, \beta = 0$



Example: Differential Drive Robot

- Therefore:

$$\begin{bmatrix} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix} \\ \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2\varphi \\ 0 \end{bmatrix}$$

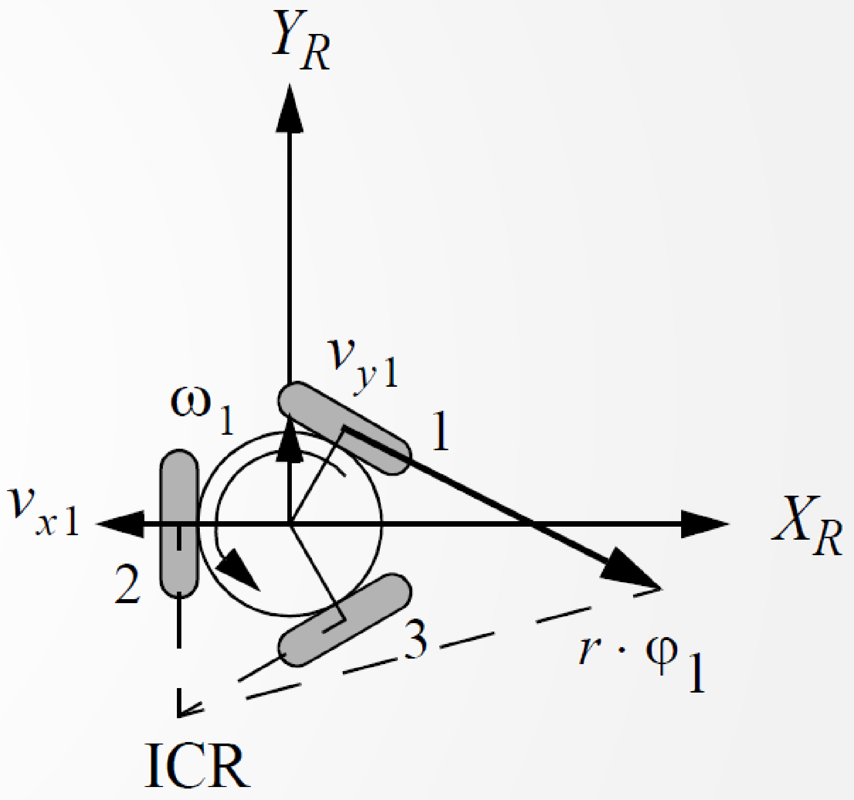


- Through inversion:

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} J_2\varphi \\ 0 \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{2l} & -\frac{1}{2l} & 0 \end{bmatrix} \begin{bmatrix} J_2\varphi \\ 0 \end{bmatrix}$$

Example: Omnidirectional Drive Robot

- Consider the omnidirection robot arrangement (and reference frames) as in the Figure.
- Assume that
  - the distance between each wheel and P is l
  - Radius of all wheels is r
- We compute the kinematic equations based on the combination of the constraints of each wheel to the robot chassis.

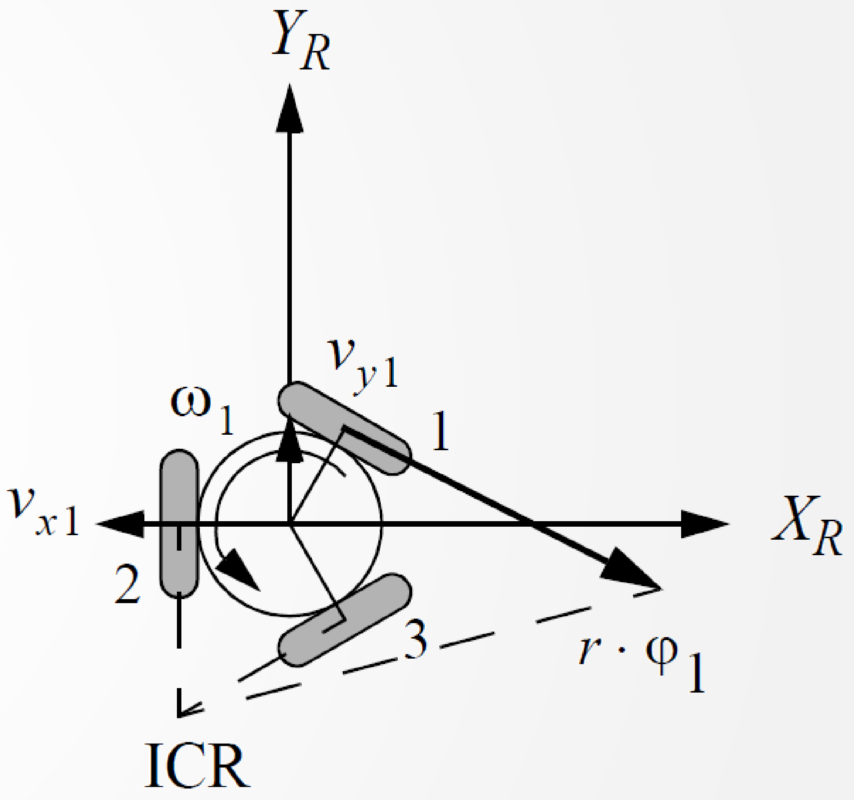


Example: Omnidirectional Drive Robot

- The value of $\dot{\xi}_I$ is can be computed as a combination of the rolling constraints of the robot's three omnidirectional wheels:

$$\dot{\xi}_I = R(\theta)^{-1} J_{1f}^{-1} J_2 \dot{\varphi}$$

- We calculate J_{1f} using the matrix elements of the rolling constraints for the Swedish wheel. Given the arrangement of the robot:
  - $\alpha_1 = \pi/3, \alpha_2 = \pi, \alpha_3 = -\pi/3$
  - $\beta = 0$
  - $\gamma = 0$



Example: Omnidirectional Drive Robot

- After calculations:

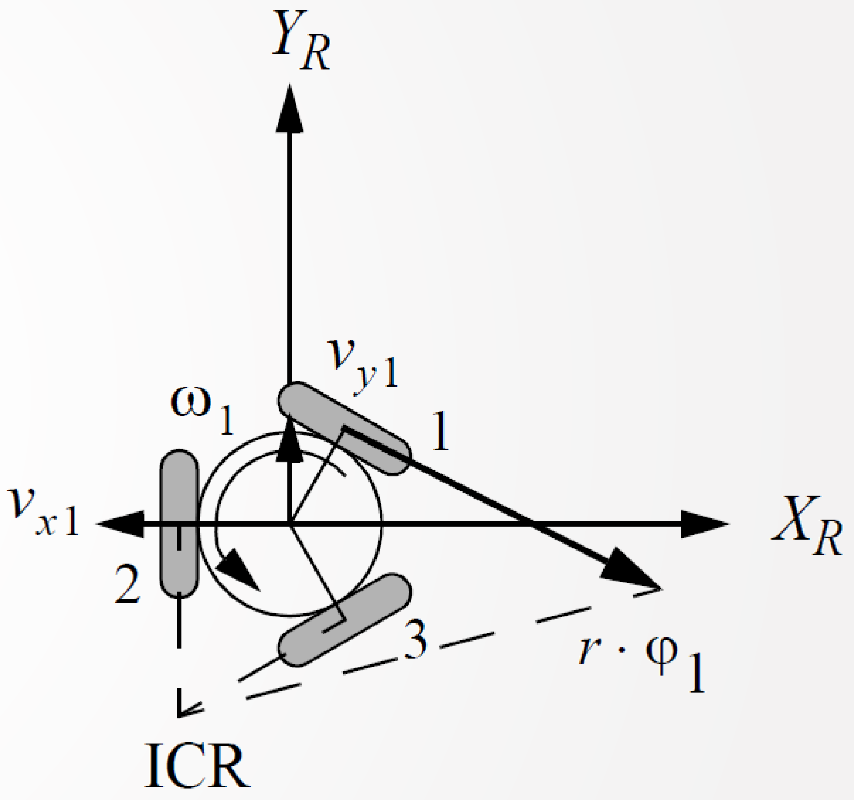
$$J_{1f} = \begin{bmatrix} \sin\frac{\pi}{3} & -\cos\frac{\pi}{3} & -l \\ 0 & -\cos\pi & -l \\ \sin-\frac{\pi}{3} & -\cos-\frac{\pi}{3} & -l \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & -l \\ 0 & 1 & -l \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & -l \end{bmatrix}$$

- Therefore:

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3l} & -\frac{1}{3l} & -\frac{1}{3l} \end{bmatrix} J_2 \dot{\varphi}$$

- Through inversion:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}} \\ -\frac{4}{3} \\ -\frac{7}{3} \end{bmatrix}$$



Mobile Robot Maneuverability

- The kinematic mobility of a robot chassis is its ability to directly move in the environment. The basic constraint limiting mobility is the rule that every wheel must satisfy its sliding constraint. Therefore, we can formally derive robot mobility by starting from:

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0$$

- In addition to instantaneous kinematic motion, a mobile robot is able to further manipulate its position over time, by steering steerable wheels. The overall maneuverability of a robot is thus a combination of the mobility available based on the kinematic sliding constraints of the standard wheels, plus the additional freedom contributed by steering and spinning the steerable standard wheels.

Degree of Maneuverability

- Defining the constraints for fixed and steerable wheels separately:

$$C_{1f}R(\theta)\dot{\xi}_I = 0 .$$

$$C_{1s}(\beta_s)R(\theta)\dot{\xi}_I = 0$$

  - For both constraints to be satisfied, the vector $R(\theta)\dot{\xi}_1$ must belong to the null space of the projection matrix $C_1(\beta_s)$, which is a combination of C_{1f} and C_{1s}.
    - The null space of $C_1(\beta_s)$ is the space N such that for any vector $n \in N$, $C_1(\beta_s)s = 0$
- The robot chassis kinematics is therefore a function of the set of independent constraints arising from all standard wheels.
  - The greater the number of constraints, and therefore the greater the rank of $C_1(\beta_s)$, the more constrained is the mobility of the robot.
  - Degree of mobility:

$$\delta_m = dimN\left[C_1(\beta_s)\right] = 3 - rank\left[C_1(\beta_s)\right]$$

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Degree of Maneuverability

- Degree of Mobility

$$\delta_m = dimN\left[C_1(\beta_s)\right] = 3 - rank\left[C_1(\beta_s)\right]$$

  - Quantifies the degrees of controllable freedom based on changes to wheel velocity.

Degree of Maneuverability

- Degree of Steerability:

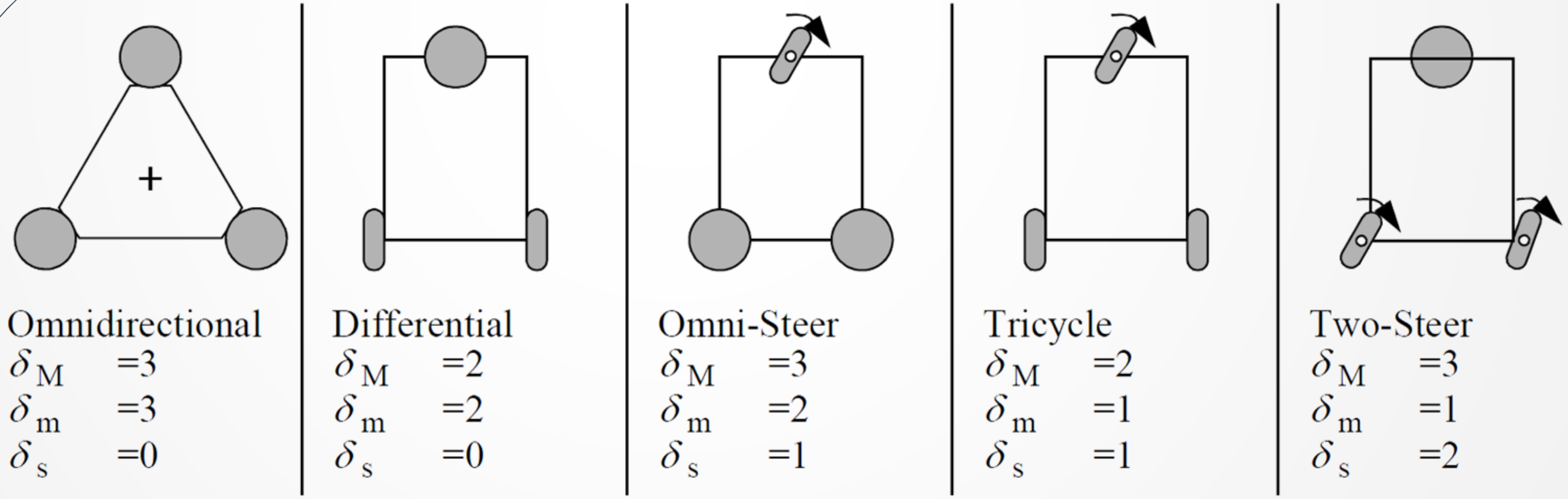
$$\delta_s = rank\left[C_{1s}(\beta_s)\right]$$

  - Quantifies the number of independently controllable steering parameters.

Degree of Maneuverability

- Degree of Maneuverability:

$$\delta_M = \delta_m + \delta_s$$



Mobile Robot Workspace

- Not all trajectories are admissible for a robot – therefore giving rise to a constraint workspace.
- *Differential Degree Of Freedom* (**DDOF**) = the number of dimensions in the velocity space of a robot is the number of independently achievable velocities or DDOF.
- A mobile robot's DDOF is equal to its degree of mobility δ_m
  - When the robot kinematic *Degrees Of Freedom* (**DOF**) are of the same number as the robot's DDOF then, this robot is **holonomic**.
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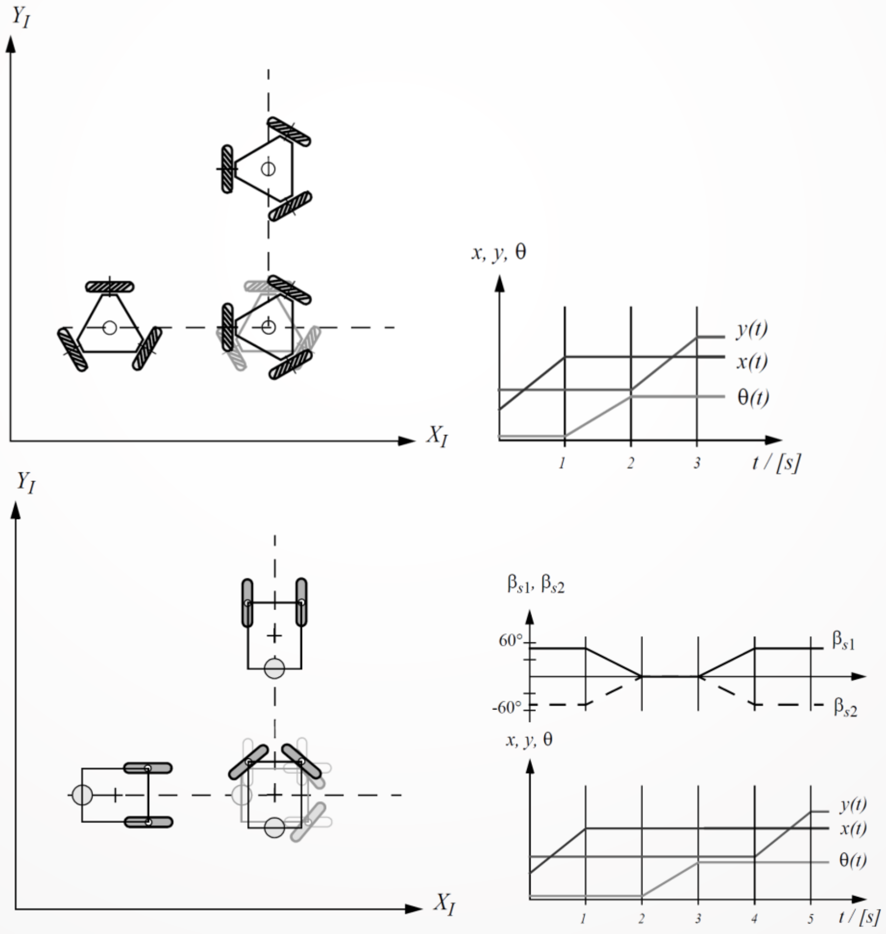
    - **A holonomic robot can admit any trajectory in the collision-free space.**
- Many broadly utilized robots are **nonholonomic**!
  - Main Example: Differential drive robot

Mobile Robot Workspace

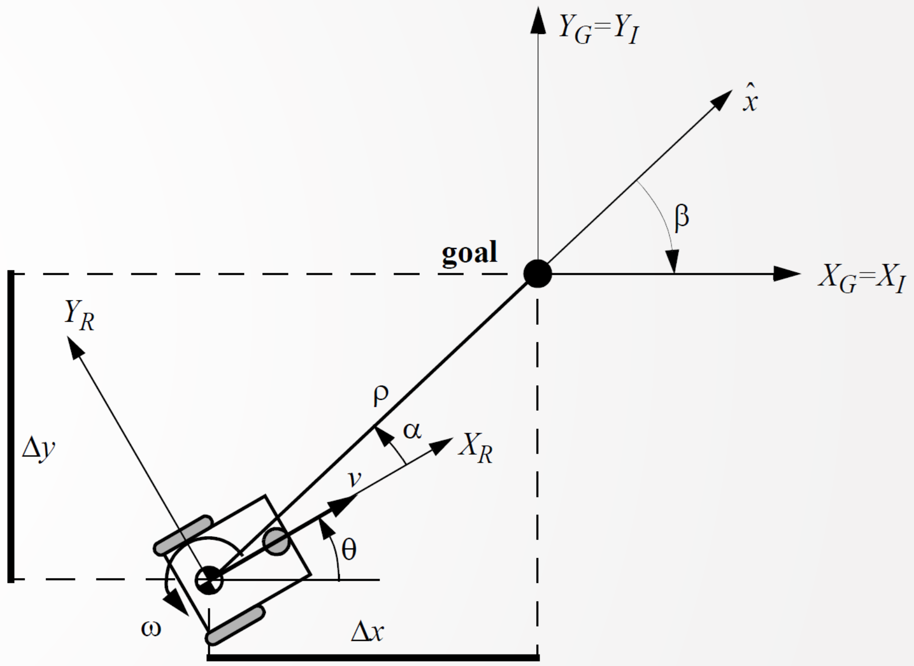
- Example of holonomic and nonholonomic robot trajectories:



Differential Drive Robot Kinematic Model

- The kinematic model of the differential drive (nonholonomic) robot takes the form:

$$^{I}\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$



- And:

$$\begin{bmatrix}\dot{\rho}\\\dot{\alpha}\\\dot{\beta}\end{bmatrix}=\begin{bmatrix}-\cos\alpha & 0\\ \frac{\sin\alpha}{\rho} & -1\\ -\frac{\sin\alpha}{\rho} & 0\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$

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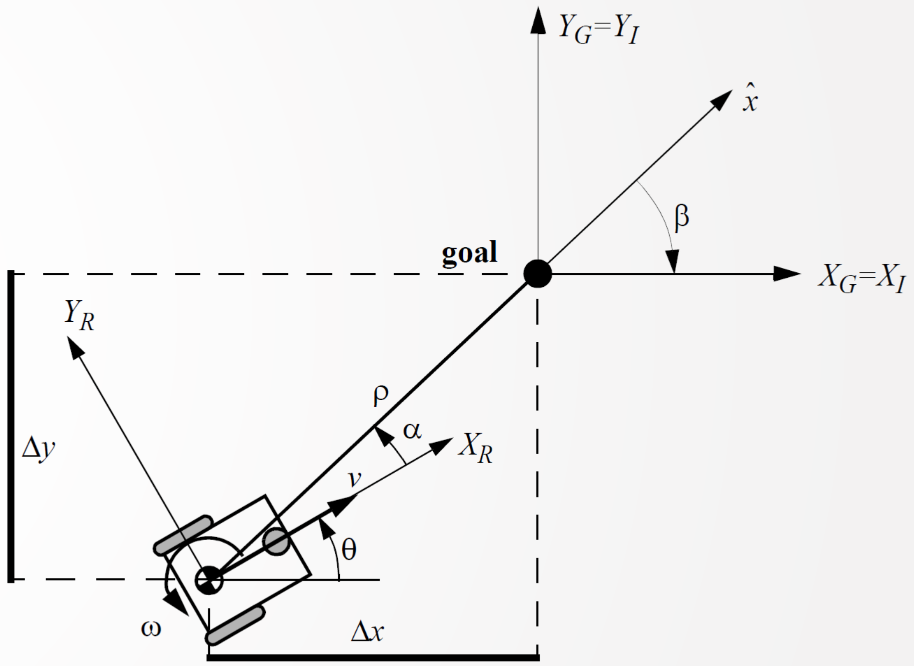
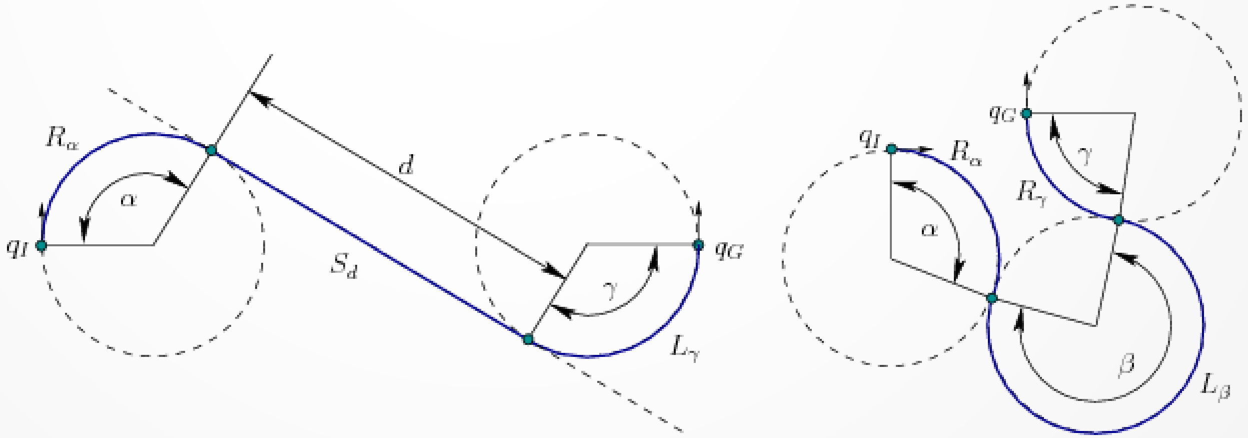


Differential Drive Robot Kinematic Model

- For differential drive robots, open-loop optimal trajectories exist: **Dubins Car** model.
- The Dubins Car model trajectories consist solely of left and right turns with maximum steering angle, and straight lines.
- The set of possible trajectories are:

{LRL, RLR, LSL, LSR, RSL, RSR}



$R_\alpha S_d L_\gamma$

$R_\alpha L_\beta R_\gamma$

Code Example

- Pythons Dubins Car Sover:
  - https://github.com/unr-arl/autonomous_mobile_robot_design_course/tree/master/python/DubinsCar
  - `Python dubins_car_example.py`

Code Example

- Indicative in-class run

Code Example

- Want to learn more? Want to get a small bonus?
  - Regenerate the Dubins Car solver in MATLAB
  - This project will give you a 2% bonus

Find out more

- http://planning.cs.uiuc.edu/node821.html
- Python? http://www.kostasalexis.com/simulations-with-simpy.html
- **Always check:** http://www.kostasalexis.com/literature-and-links1.html

Thank you!

Please ask your question!