Aerial Robotic Autonomy

> *Camera Model & Camera Calibration*

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Camera Model and Calibration

- Determine the extrinsic and intrinsic parameters of a camera sensor:
  - **Extrinsic:**
    - 3D Location and orientation of the camera
  - **Intrinsic:**
    - Focal length
    - Size of pixels

Application: Object Transfer

Application: Pose Estimation

Application: Pose Estimation

- Given a 3D model of an object, and its image (2D projection) determine the translation and rotation (pose) of the object such that when projected on the image plane it will match with the captured image frame.

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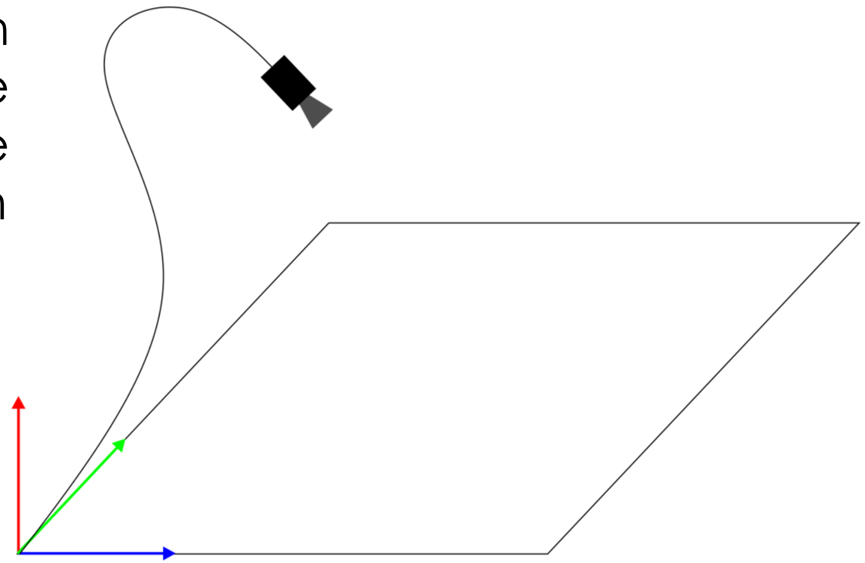
Camera Model Derivation: TFs

Defining a mathematical representation that allow us to represent the camera intrinsics and the experienced transformations from camera to world-frame scene



3D Translation

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$\mathbf{T}\mathbf{T}^{-1} = \mathbf{T}^{-1}\mathbf{T} = \mathbf{I} \Rightarrow$$

$$\mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}, \Leftrightarrow \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \mathbf{T} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}, \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \times S_x \\ Y_1 \times S_y \\ Z_1 \times S_z \end{bmatrix}$$

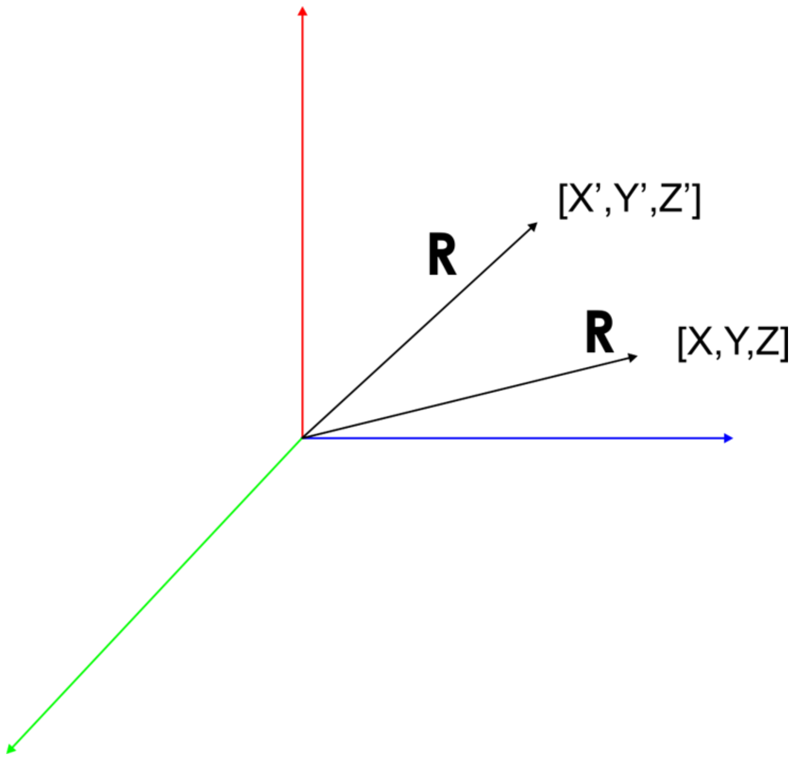
$$\mathbf{S}\mathbf{S}^{-1} = \mathbf{S}^{-1}\mathbf{S} = \mathbf{I}$$

$$\mathbf{S}^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 & 0 \\ 0 & 1/S_y & 0 & 0 \\ 0 & 0 & 1/S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \mathbf{S} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}, \ \mathbf{S} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



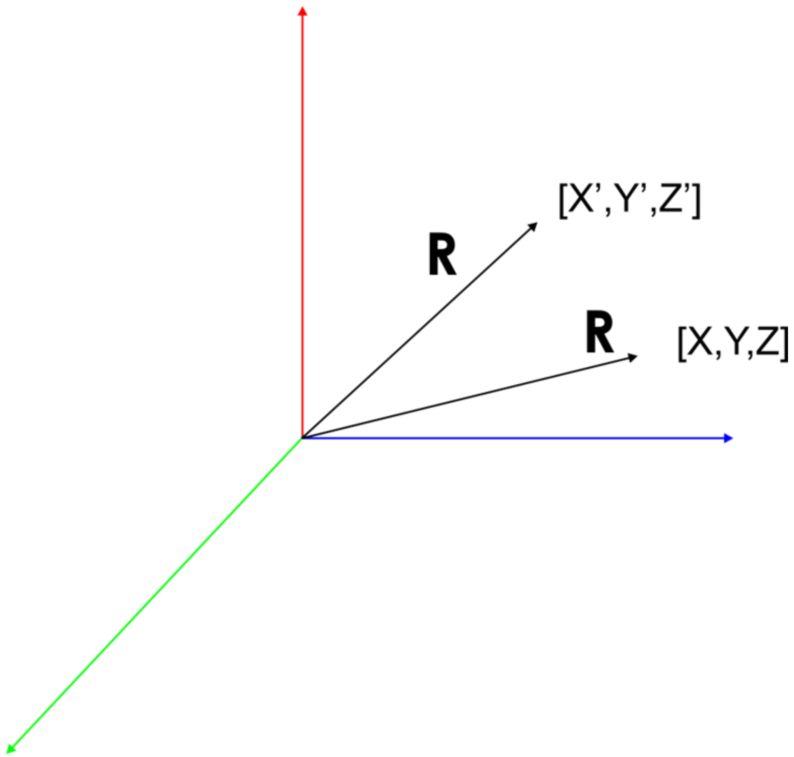
Rotation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mathbf{R}_\theta^z \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}, \quad \mathbf{R}_\theta^z = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_\theta^z(\mathbf{R}_\theta^z)^{-1} = \mathbf{R}_\theta^z(\mathbf{R}_\theta^z)^T = \mathbf{I}$$

$$r_i r_j = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$



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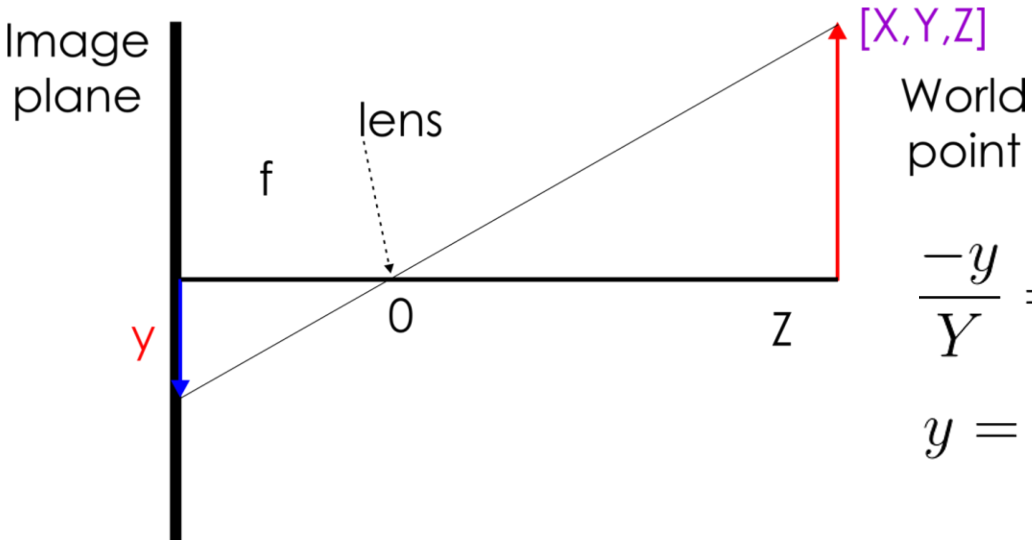
Euler Angles

$$\mathbf{R} = \mathbf{R}_Z^a \mathbf{R}_Y^\beta \mathbf{R}_X^\gamma = \begin{bmatrix} cac\beta & cas\beta s\gamma - sac\gamma & cas\beta c\gamma + sas\gamma \\ sac\beta & sas\beta s\gamma + cac\gamma & sas\beta c\gamma - cas\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

For small angles

$$\mathbf{R} = \begin{bmatrix} 1 & -a & \beta \\ a & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix}$$

Perspective Projection (origin at lens center)



$$\frac{-y}{Y} = \frac{f}{Z}$$

$$y = -\frac{fY}{Z}, \quad x = -\frac{fX}{Z}$$

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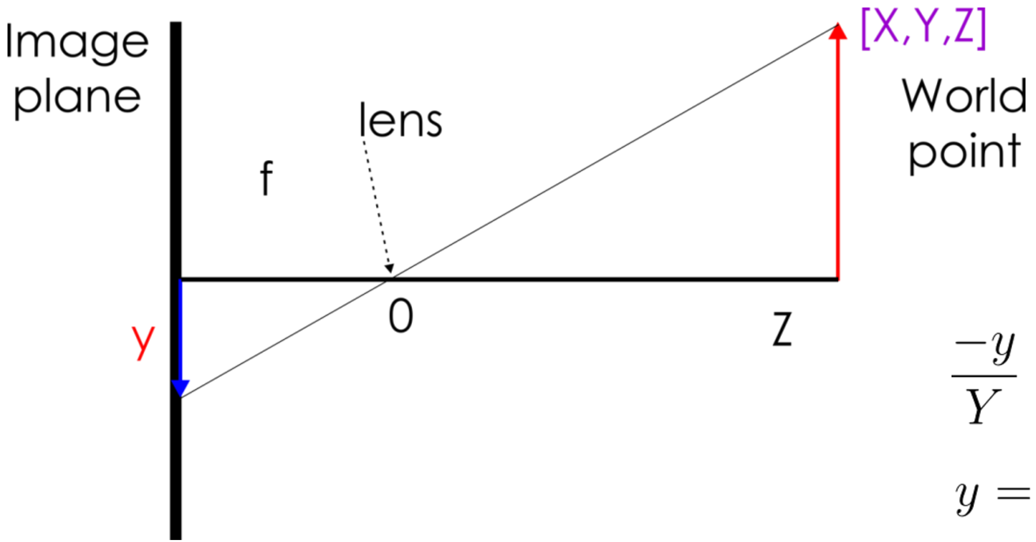
Perspective Projection (origin at image center)



$$\frac{-y}{Y} = \frac{f}{Z-f}$$

$$y = -\frac{fY}{Z-f}, \quad x = -\frac{fX}{Z-f}$$

$$y = \frac{fY}{f-Z}, \quad x = \frac{fX}{f-Z}$$

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Perspective Formulation

Image coordinates

$$\begin{bmatrix} x \\ y \\ \\ \\ \end{bmatrix} = \begin{bmatrix} & & \\ & ? & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

World coordinates

$$[X, Y, Z] \rightarrow [kX, kY, kZ, k]$$

Homogeneous transformation

$$[C_{h1}, C_{h2}, C_{h3}, C_{h4}] \rightarrow [\frac{C_{h1}}{C_{h4}}, \frac{C_{h2}}{C_{h4}}, \frac{C_{h3}}{C_{h4}}]$$

Inverse transformation

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 1 \end{bmatrix}$$

Perspective Formulation

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}} = \frac{kX}{k - \frac{kZ}{f}} = \frac{fX}{f - Z}$$

$$y = \frac{C_{h2}}{C_{h4}} = \frac{kY}{k - \frac{kZ}{f}} = \frac{fY}{f - Z}$$

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix} = \begin{bmatrix} kX \\ kY \\ kZ \\ k - \frac{kZ}{f} \end{bmatrix}$$

Camera Model

- Camera is at the origin of the world coordinates first
- Then translated by some amount (G)
- Then rotated around Z axis in counter clockwise direction
- Then rotated again around X in counter clockwise direction
- Then translated by C

$$\mathbf{C}_h = \mathbf{PCR}^X_{-\phi}\mathbf{R}^Z_{-\theta}\mathbf{GW}_h$$

Since we are moving the camera instead of object we need to use inverse transformations

Camera Model

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}, \mathbf{R}^Z_{-\theta} = \begin{bmatrix} c\theta & s\theta & 0 & 0 \\ -s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}^X_{-\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\phi & s\phi & 0 \\ 0 & -s\phi & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & -r_1 \\ 0 & 1 & 0 & -r_2 \\ 0 & 0 & 1 & -r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Model

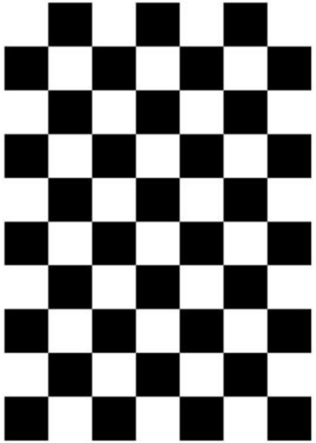
$$\mathbf{C}_h = \mathbf{P}\mathbf{C}\mathbf{R}^X_{-\phi}\mathbf{R}^Z_{-\theta}\mathbf{G}\mathbf{W}_h$$

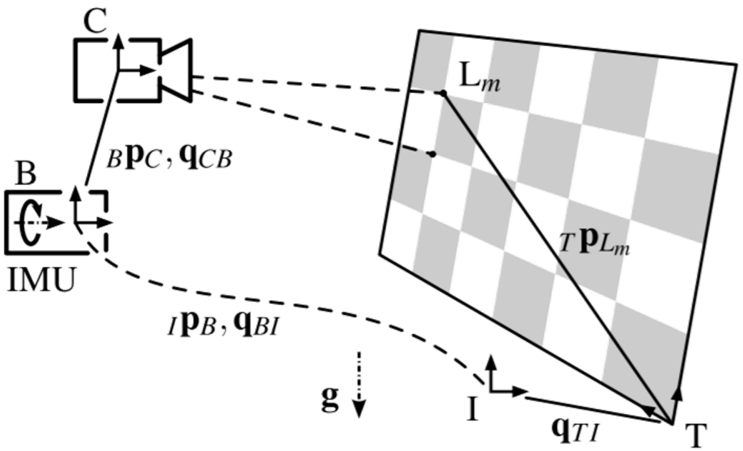
$$x = f\frac{(X - X_0)c\theta + (Y - Y_0)s\theta - r_1}{-(X - X_0)s\theta s\phi + (Y - Y_0)c\theta s\phi - (Z - Z_0)c\phi + r_3 + f}$$

$$y = f\frac{(X - X_0)s\theta c\phi + (Y - Y_0)c\theta c\phi + (Z - Z_0)s\phi - r_2}{-(X - X_0)s\theta s\phi + (Y - Y_0)c\theta s\phi - (Z - Z_0)c\phi + r_3 + f}$$

Camera Model

- How to determine camera matrix?
- Select some known 3D points (X,Y,Z), and find their corresponding image points (x,y)
- Solve for camera matrix elements using Least Squares Fit





Camera Model

$$\mathbf{C}_h = \mathbf{PCR}^X_{-\phi}\mathbf{R}^Z_{-\theta}\mathbf{GW}_h$$

$$\mathbf{C}_h = \mathbf{AW}_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$
$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$
$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

Ch3 is not needed, we have 12 unknowns

Camera Model

$$\mathbf{C}_h = \mathbf{P}\mathbf{C}\mathbf{R}^X_{-\phi}\mathbf{R}^Z_{-\theta}\mathbf{G}\mathbf{W}_h$$

$$\mathbf{C}_h = \mathbf{A}\mathbf{W}_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

Solve for these unknowns

$$\boxed{\begin{aligned} Ch_1 &= a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x \\ Ch_2 &= a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y \\ Ch_4 &= a_{41}X + a_{42}Y + a_{43}Z + a_{44} \end{aligned}}$$

Ch3 is not needed, we have 12 unknowns

Camera Model

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4x$$
$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4y$$
$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

Substitute *Ch4*

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$
$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

Camera Model

single point

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

$$a_{11}X_1 + a_{12}Y_1 + a_{13}Z_1 + a_{14} - a_{41}X_1x_1 - a_{42}Y_1x_1 - a_{43}Z_1x_1 - a_{44}x_1 = 0$$

$$a_{11}X_2 + a_{12}Y_2 + a_{13}Z_2 + a_{14} - a_{41}X_2x_2 - a_{42}Y_2x_2 - a_{43}Z_2x_2 - a_{44}x_2 = 0$$

$$\vdots$$

$$a_{11}X_n + a_{12}Y_n + a_{13}Z_n + a_{14} - a_{41}X_nx_n - a_{42}Y_nx_n - a_{43}Z_nx_n - a_{44}x_n = 0$$

$$a_{21}X_1 + a_{22}Y_1 + a_{23}Z_1 + a_{24} - a_{41}X_1y_1 - a_{42}Y_1y_1 - a_{43}Z_1y_1 - a_{44}y_1 = 0$$

$$a_{21}X_2 + a_{22}Y_2 + a_{23}Z_2 + a_{24} - a_{41}X_2y_2 - a_{42}Y_2y_2 - a_{43}Z_2y_2 - a_{44}y_2 = 0$$

$$\vdots$$

$$a_{21}X_n + a_{22}Y_n + a_{23}Z_n + a_{24} - a_{41}X_ny_n - a_{42}Y_ny_n - a_{43}Z_ny_n - a_{44}y_n = 0$$

n points
2n equations
12 unknowns

Camera Model

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\
X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2Z_2 & -x_2 \\
 & & & & & \vdots & & & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 & -y_1 \\
0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 & -y_2 \\
 & & & & & \vdots & & & & & & \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nZ_n & -y_n
\end{bmatrix}
\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ a_{44} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

This is a homogeneous system, therefore has no unique solution

$$CP = 0$$

Select: $a_{44} = 1$

Camera Model

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2Z_2 & -x_2 \\ & & & & & \vdots & & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 & -y_1 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 & -y_2 \\ & & & & & \vdots & & & & & & \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nZ_n & -y_n \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ a_{44} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This is a homogeneous system, therefore has no unique solution

$$CP = 0$$

Select: $a_{44} = 1$

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Camera Model

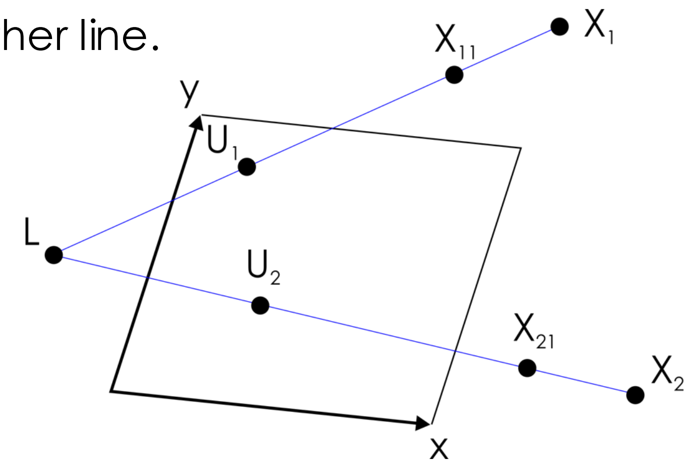
$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 \\
X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2Z_2 \\
 & & & & & & & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 \\
0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 \\
 & & & & & & & & & & \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nZ_n
\end{bmatrix}
\begin{bmatrix}
a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43}
\end{bmatrix}
=
\begin{bmatrix}
x_1 \\ x_2 \\ \\ \\ x_n \\ y_1 \\ y_2 \\ \\ \\ y_n
\end{bmatrix}
$$

Pseudo inverse
Least squares fit

$$
\mathbf{DQ} = R \Rightarrow
$$
$$
\mathbf{D}^T\mathbf{Q} = \mathbf{D}^T\mathbf{R} \Rightarrow
$$
$$
\mathbf{Q} = (\mathbf{D}^T\mathbf{D})^{-1}\mathbf{D}^T\mathbf{R}
$$

Finding Camera Location

- Take one 3D point X_1 and find its image homogeneous coordinates.
- Set the third component of homogeneous coordinates to zero, find corresponding World coordinates of that point, X_{11}
- Connect X_1 and X_{11} to get a line in 3D
- Repeat this for another 3D point X_2 and find another line.
- Two lines will intersect at the location of camera.



Camera Location

$$\mathbf{C}_h = \mathbf{A}\mathbf{W}_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{U}_1 = \mathbf{A}\mathbf{X}_1$$
$$\mathbf{U}_1' = [Ch_1\ Ch_2\ 0\ Ch_4]$$
$$\mathbf{X}_{11} = \mathbf{A}^{-1}\mathbf{U}_1'$$

$$\mathbf{U}_2 = \mathbf{A}\mathbf{X}_2$$
$$\mathbf{U}_2' = [Ch_1\ Ch_2\ 0\ Ch_4]$$
$$\mathbf{X}_2 = \mathbf{A}^{-1}\mathbf{U}_2'$$

Camera Orientation

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \mathbf{A} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}}$$

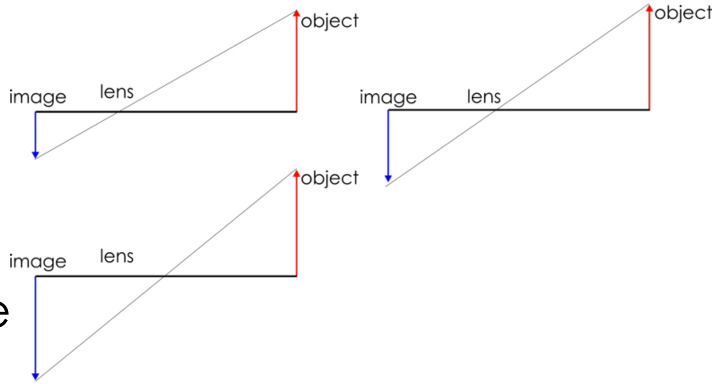
$$y = \frac{C_{h2}}{C_{h4}}$$

Only time the image wil be formed at infinity if $Ch4=0$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ 0 \end{bmatrix} = \mathbf{A} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$a_{41}X + a_{42}Y + a_{43}Z + a_{44} = 0$$

This is the equation of a plane going through the lens, which is parallel to image plane.



Historical Application

$$\mathbf{M} = \begin{bmatrix} 0.17237 & -0.15879 & 0.01879 & 274.943 \\ 0.131132 & 0.112747 & 0.2914 & 258.686 \\ 0.000346 & 0.0003 & 0.00006 & 1 \end{bmatrix}$$

- **Camera location:** Intersection of California and Mason streets, at an elevation of 435ft asl. The camera was oriented at an angle of 8o above the horizon with fsx = 495, fsy = 560.

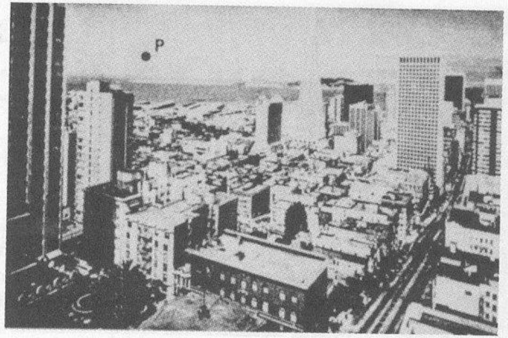


FIGURE 8 PHOTOGRAPH OF SAN FRANCISCO

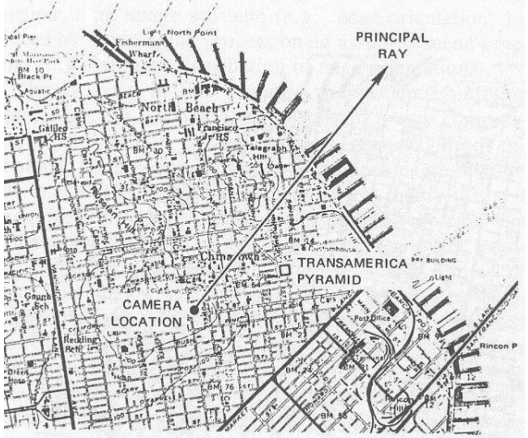


FIGURE 9 MAP OF SAN FRANCISCO

Camera Parameters

- **Extrinsic parameters**
  - Parameters that define the location and orientation of the camera reference frame with respect to a known world reference frame
    - 3D translation vector
    - A 3x3 rotation matrix
- **Intrinsic parameters**
  - Parameters necessary to link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame
    - Perspective projection (focal length)
    - Transformation between camera frame coordinates and pixel coordinates

Code Examples and Tasks

- Conduct Camera Calibration
  - MATLAB: https://www.mathworks.com/help/vision/ug/single-camera-calibrator-app.html
    - https://www.mathworks.com/videos/camera-calibration-with-matlab-81233.html
    - Collected matlab docs: https://github.com/unr-arl/autonomous_mobile_robot_design_course/tree/master/matlab/localization-mapping/mono_camera_calibration
  - ROS:
    - Monocular camera: http://wiki.ros.org/camera_calibration
    - Stereo camera: http://wiki.ros.org/camera_calibration/Tutorials/StereoCalibration
    - Get a USB camera working with ROS: http://wiki.ros.org/usb_cam
      - Indicative launch file: https://github.com/unr-arl/autonomous_mobile_robot_design_course/blob/master/ROS/sensors/camera/launch_webcam.launch

Find out more

- Chapter 1, “Fundamentals of computer vision”, Mubarak Shah
- Strat, Thomas M. "Recovering the camera parameters from a transformation matrix." Readings in Computer Vision (1987): 93-100. APA
- Zhang, Zhengyou. "A flexible new technique for camera calibration." IEEE Transactions on pattern analysis and machine intelligence 22.11 (2000): 1330-1334.
- Zhang, Zhengyou. "Flexible camera calibration by viewing a plane from unknown orientations." Computer Vision, 1999. The Proceedings of the Seventh IEEE International Conference on. Vol. 1. Ieee, 1999.
- Furgale, Paul, Joern Rehder, and Roland Siegwart. "Unified temporal and spatial calibration for multi-sensor systems." 2013 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE, 2013.