Aerial Robotic Autonomy

> *Geometric Tracking Control on SE(3)*

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Introduction

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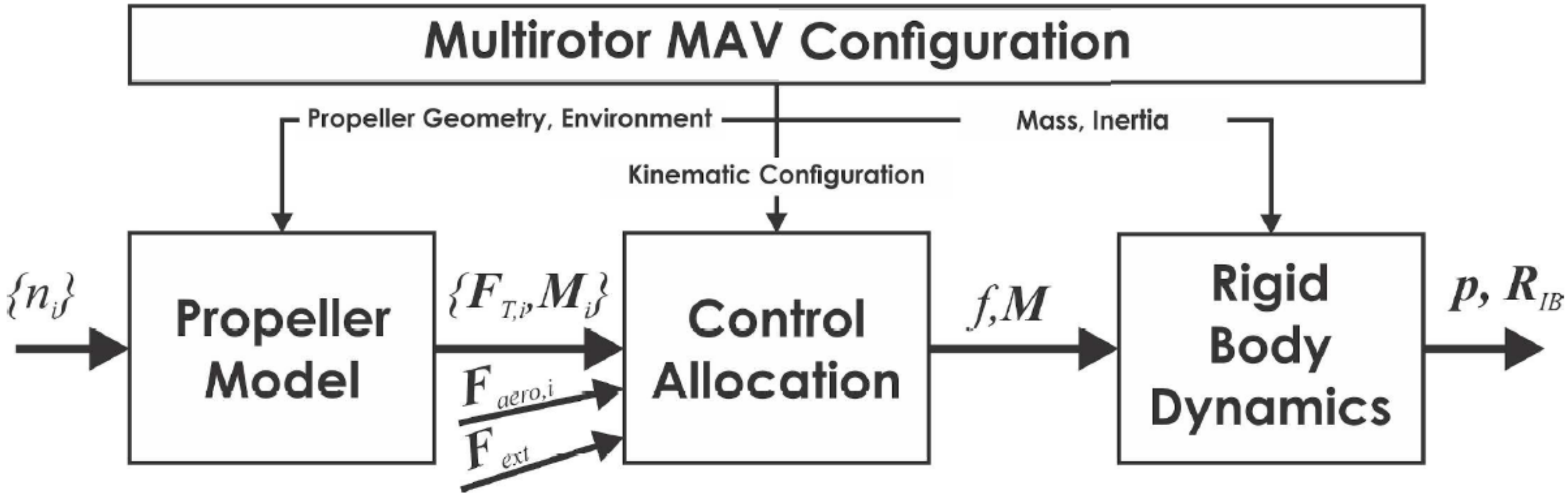
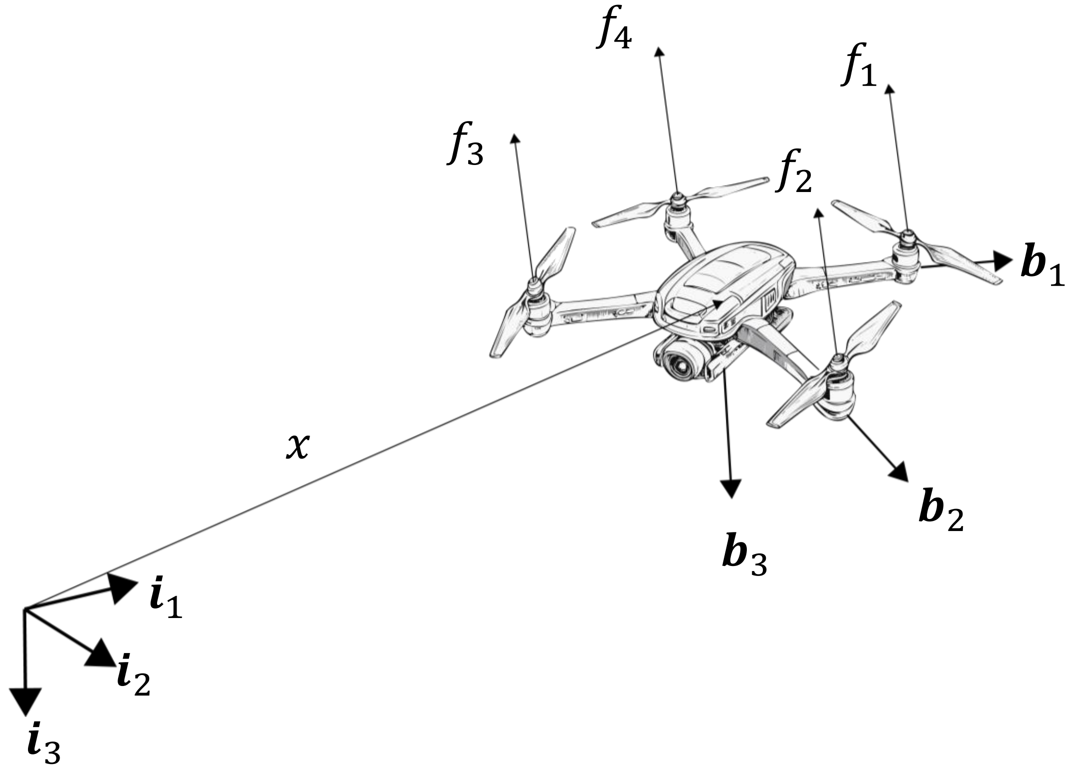
- New control approach uses simple fixed-gain formulation but combined with error formulation on SE(3).
- It thus exhibits desirable, almost global, performance properties.

MAV Modeling Summary



MAV Modeling Summary



MAV Modeling Summary

For each propeller i the generated thrust and moment take the simplified form (n_i rotor speed, $k_n, k_m > 0$ constants, e_z unit vector in the z direction)

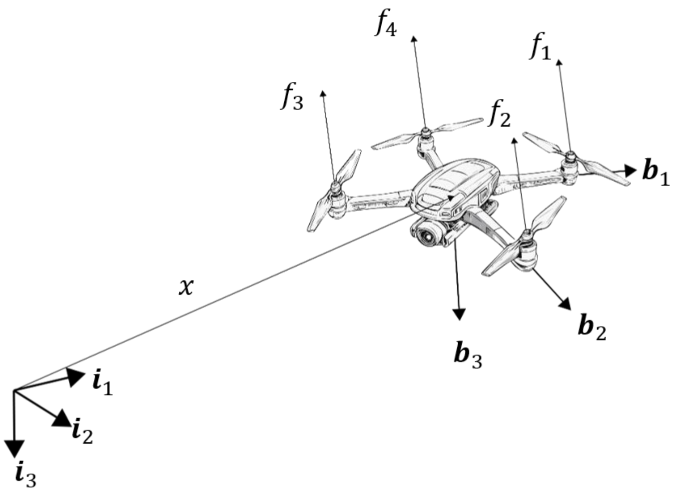
$$\begin{aligned} \mathbf{F}_{T,i} &= k_n n_i^2 \mathbf{e}_z \\ \mathbf{M}_i &= (-1)^{i-1} k_m \mathbf{F}_{T,i} \end{aligned}$$

Lamped effect of phenomena such as blade flapping ($\boldsymbol{K}_{drag} = \mathbf{diag}(k_D, k_D, 0), k_D > 0$ and $f_{T,i}$ is the z component of the i-th thrust force)

$$\mathbf{F}_{aero,i} = f_{T,i} \mathbf{K}_{drag} \mathbf{R}_{IB}^T \mathbf{v}$$

MAV Modeling Summary

Considering explicitly the case of a quadrotor:



x - the location of the center of mass.
v - the velocity of the center of mass in the inertial frame.
d - the distance from the center of mass to the center of each rotor in the $\boldsymbol{b}_1, \boldsymbol{b}_2$ plane
f_i - the thrust generated by the i-th propeller along the $-\boldsymbol{b}_3$ axis.
τ_i - the torque generated by the i-th propeller about the $\boldsymbol{b}_3$ axis.
f - the total thrust, i.e., $f = \Sigma_{i=1}^{4} f_i$
M - the total moment in the body-fixed frame

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MAV Modeling Summary

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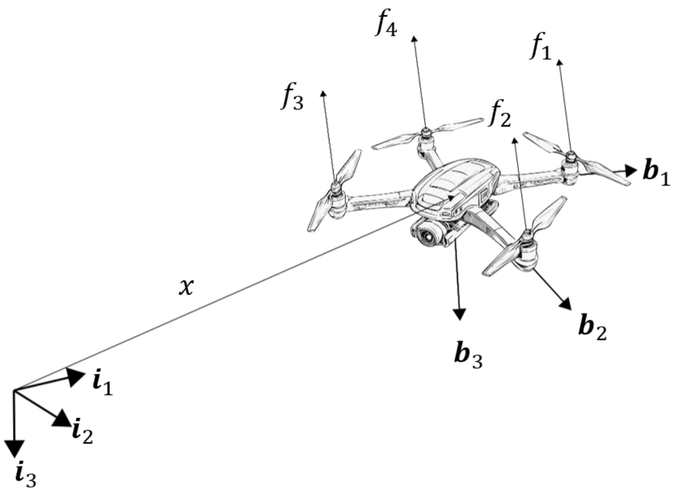


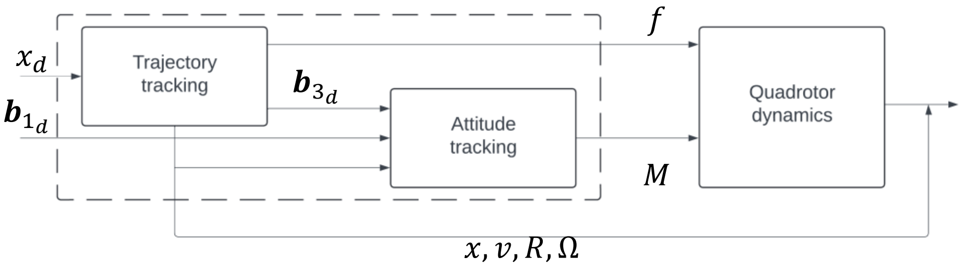
Simplified equations of motion representation

$$\dot{x} = v$$
$$m\dot{v} = mge_3 - fRe_3$$
$$\dot{R} = R\widehat{\Omega}$$
$$J\dot{\Omega} + \Omega \times J\Omega = M$$

where the hat map is defined by the condition that $\hat{x}y = x \times y$ for all x,y in $\mathbb{R}^3$

Geometric Tracking on SE(3)

Control architecture



- Translational dynamics controlled by the total thrust $-fRe_3$
- For a given translational command x_d we pick the total thrust and the desired $\boldsymbol{b}_{3_d}$
- Once the desired direction of the third body-fixed frame $\boldsymbol{b}_{3_d}$ is selected then there is one remaining degree of freedom in selecting the attitude $R_d \in SO(3)$ which corresponds to the heading direction and is determined by $\boldsymbol{b}_{1_d}$ meaning the first body axis of the desired direction.,

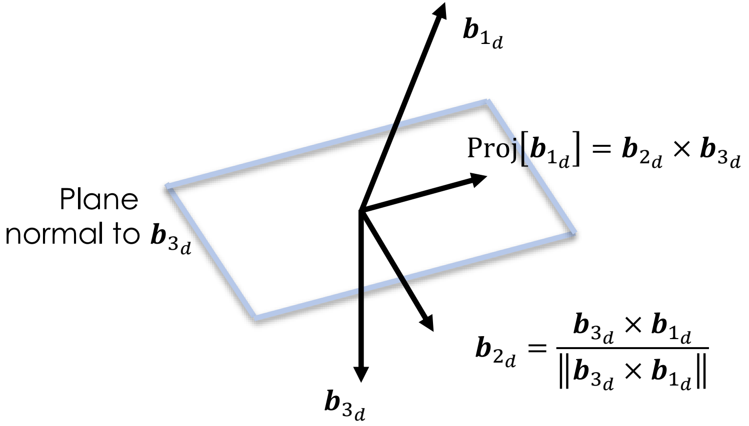
Geometric Tracking on SE(3)

- Assume that b_{1_d} is not parallel b_{3_d} to obtain the complete desired attitude $R_d = \left[\boldsymbol{b}_{2_d} \times \boldsymbol{b}_{3_d}, \boldsymbol{b}_{2_d}, \boldsymbol{b}_{3_d}\right] \in SO(3)$, with $\boldsymbol{b}_{2_d} = (\boldsymbol{b}_{3_d} \times \boldsymbol{b}_{1_d}) / \left\| \boldsymbol{b}_{3_d} \times \boldsymbol{b}_{1_d} \right\|$
- The control moment shall be selected to track this desired attitude.

Geometric Tracking on SE(3)

- 4-dimensional control inputs are designed to follow a 3-dimensional translational command plus a 1-dimensional heading direction.
- Accordingly, the controller guarantees that
  - $x \to x_d$
  - $\mathrm{Proj}[\boldsymbol{b}_1] \to \mathrm{Proj}[\boldsymbol{b}_{1_d}]$

  as $t \to \infty$ while $\mathrm{Proj}[\cdot]$ is the normalized projection onto the plane orthogonal to $\boldsymbol{b}_{3_d}$



Tracking Errors

Position & Velocity errors

$$e_x = x - x_d$$
$$e_v = v - v_d$$

Tracking Errors

Attitude & attitude velocity errors

The SO(3) error function is selected to be

$$\Psi(R, R_d) = \frac{1}{2}\mathrm{tr}[I - R_d^T R]$$

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Then when the variation of the rotation matrix is expressed as $\delta R = \hat{R}\hat{\eta}$ the derivative of the error function takes the form

$$D_R\Psi(R, R_d) \cdot R\hat{\eta} = -\frac{1}{2}\mathrm{tr}[R_d^T R\hat{\eta}] = \frac{1}{2}\left(R_d^T R - R^T R_d\right)^{\vee}\eta$$

where the vee map (∨) is the inverse of the hat map.

Tracking Errors

Attitude & attitude velocity errors

Accordingly, the attitude tracking error e_R takes the form

$$e_R = \frac{1}{2}\left(R_d^T R - R^T R_d\right)^{\vee}$$

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The tangent vectors $\dot{R} \in T_R SO(3)$ and $\dot{R}_d \in T_{R_d} SO(3)$ cannot be directly compared with each other as they lie on different tangent spaces. We thus transform $\dot{R}_d$ into a vector in $T_R SO(3)$ and compare that with $\dot{R}$

$$\dot{R} - \dot{R}_d\left(R_d^T R\right) = R\widehat{\Omega} - R_d \widehat{\Omega}_d R_d^T R = R(\Omega - R^T R_d \Omega_d)^{\wedge}$$

and choose the tracking error for the angular velocity as

$$e_\Omega = \Omega - R^T R_d \Omega_d$$

Tracking Errors

Attitude & attitude velocity errors

Note that the term

$$e_\Omega = \Omega - R^T R_d \Omega_d$$

Can be shown to be the angular velocity of the rotation matrix $R_d^T R$ represented in the body fixed frame.

This holds as

$$\frac{d}{dt}\left(R_d^T R\right) = (R_d^T R)\hat{e}\_\Omega$$

Tracking Controllers

For positive constants k_x, k_v, k_R, k_Ω, assume that

$$\|-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d\| \neq 0$$

Then we defined the desired $\boldsymbol{b}_{3_d}$ as

$$\boldsymbol{b}_{3_d} = -\frac{-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d}{\|-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d\|}$$

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Furthermore, it is also assumed that $\boldsymbol{b}_{1_d}$ is not parallel to $\boldsymbol{b}_{3_d}$.

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Furthermore, it is also assumed that $\boldsymbol{b}_{1_d}$ is not parallel to $\boldsymbol{b}_{3_d}$.

Then, the desired attitude is given by

$$R_d = \left[\boldsymbol{b}_{2_d} \times \boldsymbol{b}_{3_d}, \boldsymbol{b}_{2_d}, \boldsymbol{b}_{3_d}\right] \in SO(3),$$

where

$$\boldsymbol{b}_{2_d} = (\boldsymbol{b}_{3_d} \times \boldsymbol{b}_{1_d}) / \left\|\boldsymbol{b}_{3_d} \times \boldsymbol{b}_{1_d}\right\|$$

Tracking Controllers

Given that the desired trajectory satisfies

$$\|-mge_3 + m\ddot{x}_d\| < B$$

for some given constant B term the control inputs f, M take the form

$$f = -(-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d) \cdot Re_3$$
$$M = -k_R e_R - k_\Omega e_\Omega + \Omega \times J\Omega - J(\hat{\Omega} R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d)$$

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The moments M above represent a tracking controller on $SO(3)$

The term $(-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d)$ represents a tracking controller on $\mathbb{R}^3$ and given the selection of $\boldsymbol{b}_{3_d}$ and f the translational tracking error shall converge to zero when the attitude tracking error is zero.

Code Examples and Tasks

- https://github.com/fdcl-gwu/uav_geometric_control
- https://github.com/ethz-asl/rotors_simulator
  - https://github.com/ethz-asl/rotors_simulator/blob/master/rotors_control/include/rotors_control/lee_position_controller.h

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Find out more

- Lee, T., Leok, M. and McClamroch, N.H., 2010, December. Geometric tracking control of a quadrotor UAV on SE (3). In 49th IEEE conference on decision and control (CDC) (pp. 5420-5425). IEEE.
- Gamagedara, K., Bisheban, M., Kaufman, E. and Lee, T., 2019, July. Geometric controls of a quadrotor uav with decoupled yaw control. In 2019 American Control Conference (ACC) (pp. 3285-3290). IEEE.
- Lee, T., 2012. Exponential stability of an attitude tracking control system on SO (3) for large-angle rotational maneuvers. Systems & Control Letters, 61(1), pp.231-237.