Airborne LiDAR System (ALS)
Ranging and Georeferencing

ROD PICKENS
CHIEF SCIENTIST, CNS
SIERRA NEVADA CORPORATION

FRIDAY MARCH 31, 2017
Topics to cover

- Remote Sensing with LiDAR
- Airborne LiDAR Systems
- Ranging methods for full-waveform LiDAR
- Georeferencing LiDAR point cloud
- Conclusion
- References
Remote Sensing with LiDAR
LiDAR data
Airborne LiDAR System: Overview

- **System Controller**
  - **Position and Orientation (POS)**: GPS, IMU
  - **Ranging Subsystem**: Laser Tx, Laser Rx
  - **Pointing System**
  - **Storage**
  - **Processing**
    - Range Extraction
    - Georegistration

Scene

*NGA.SIG.0004
Pointing

System Controller

Position and Orientation (POS)
- GPS
- IMU

Ranging Subsystem
- Laser Tx
- Laser Rx

Storage

Processing
- Range Extraction
- Georegistration

Scene

*NGA.SIG.0004
LiDAR Pointing Mechanisms

LiDAR

Scanner: Flying Spot

Scannerless: Time-of-Flight Camera

Avalanche Photo Diode: Photo by Radovan Blazek

Advanced Scientific Concepts: FLASH Lidar
Range Extraction

System Controller

Position and Orientation (POS)
- GPS
- IMU

Ranging Subsystem
- Laser Tx
- Laser Rx

Pointing System

Storage

Processing
- Range Extraction
- Georegistration

Scene

*NGA.SIG.0004*
Approaches to Range Extraction

Discrete returns (hardware based)

Full-waveform returns (software based)

Figure 1.05: Multiple lidar returns can be generated from a single emitted pulse. Depending on the detection capabilities of the lidar sensor, two or more of these returns may be recorded as data points.

SOURCE: ASPRS

Figure 1.07: In a waveform lidar, the entire return pulse is digitized and recorded. In a discrete multiple-return lidar, only the peaks would be recorded.

SOURCE: ASPRS
Principles of ToF Imaging

Pulsed Modulation

- Measure distance to a 3D object by measuring the absolute time a light pulse needs to travel from a source into the 3D scene and back, after reflection.

- Speed of light is constant and known, \( c = 3 \cdot 10^8 \text{m/s} \)
Processing System Controller

Position and Orientation (POS)
- GPS
- IMU

Ranging Subsystem
- Laser Tx
- Laser Rx

Pointing System

Storage

Scene

Processing
- Range Extraction
- Georegistration

*NGA.SIG.0004
Full-waveform Range Extraction
Full-waveform Range Extraction

Discrete returns (hardware based)

Full-waveform returns (software based)

Figure 1.05: Multiple lidar returns can be generated from a single emitted pulse. Depending on the detection capabilities of the lidar sensor, two or more of these returns may be recorded as data points. 

SOURCE: ASPRS

Figure 1.07: In a waveform lidar, the entire return pulse is digitized and recorded. In a discrete multiple-return lidar, only the peaks would be recorded.

SOURCE: ASPRS
Some Methods of Range Extraction

- **Gaussian Decomposition**
  - Hofton et al. 2000, Persson et al. 2005, ...

- **Expectation-Maximization Deconvolution**
  - Parrish et al 2007, Figueiredo and Nowak 2003

- **Wiener Deconvolution and Decomposition**
  - Jutzi and Silla 2006

- **Other methods**
  - Matched filtering
  - B-Splines Approach
    - Roncat et al. 2010
  - Average Square Difference Function
    - Wagner et al 2007
Gaussian Decomposition
Each LiDAR waveform is a linear combination of Gaussian components.

\[
\hat{y}(t) = \sum_{i=1}^{N} \alpha_i \exp \left\{ -\frac{1}{2\sigma_i^2} (t - \mu_i)^2 \right\}
\]

\(\alpha_i = \text{amplitude of } ith \text{ component}\)

\(\sigma_i = \text{width of } ith \text{ component}\)

\(\mu_i = \text{position of } ith \text{ component}\)

Sources: References 4 and 5
Functional Diagram of Gaussian Decomposition

Hofton: Reference 3 and 4
Expectation Maximization
Deconvolution
Deconvolution: Determine $x[n]$

Given input $y[n]$ which is the received full waveform:

$$y[n] = h[n] * x[n] + \eta[n]$$

where

- $x[n]$ is the full waveform representation of scene
- $h[n]$ is system impulse response (optics, electronics, atmosphere)
- $\eta[n]$ is white Gaussian noise

estimate via deconvolution

$$x[n]$$

which is the signal of interest.
Expectation Maximization (1/5)

Maximum Likelihood

\[ \hat{\theta} = \arg\max_{\theta} \left[ \sum_{i=1}^{I} \log \left[ \Pr(x_i | \theta) \right] \right] \]

Expectation maximization (hidden variables \( h_i \))

\[ \hat{\theta} = \arg\max_{\theta} \left[ \sum_{i=1}^{I} \log \left[ \int \Pr(x_i, h_i | \theta) \, dh_i \right] \right] \]

\[ \Pr(x_i | \theta) = \int \Pr(x_i, h_i | \theta) \, dh_i \]
Apply EM to Mixture of Gaussians

Figure 7.7 Mixture of Gaussians as a marginalization. The mixture of Gaussians can also be thought of in terms of a joint distribution $Pr(x, h)$ between the observed variable $x$ and a discrete hidden variable $h$. To create the mixture density we marginalize over $h$. The hidden variable has a straightforward interpretation: it is the index of the constituent normal distribution.

Source: Prince
\[ \hat{\theta} = \arg\max_{\theta} \left[ \sum_{i=1}^{l} \log \left[ \int \Pr(x_i, h_i | \theta) \, dh_i \right] \right] \]

Set a lower bound to the above log likelihood, \( A \), as follows:

\[ \sum_{i=1}^{l} q_i(h_i) \log \left[ \int \frac{\Pr(x_i, h_i | \theta)}{q_i(h_i)} \, dh_i \right] \leq \sum_{i=1}^{l} \log \left[ \int \Pr(x_i, h_i | \theta) \, dh_i \right] \]
Set $q_i(h_i)$ with hidden parameters $h_i$ as follows

E-step: 

$$q_i(h_i) = P(h_i|x_i, \theta^{[t]}) = \frac{Pr(x_i|h_i, \theta^{[t]})Pr(h_i|\theta^{[t]})}{Pr(x_i)}$$

and maximize for $\theta$ as follows

M-step: 

$$\hat{\theta}^{[t+1]} = \arg\max_{\theta} \sum_{i=1}^{I} q_i(h_i) \log \left[ \int Pr(x_i, h_i|\hat{\theta}^{[t]}) \, dh_i \right]$$
Range Extraction with EM (5/5)

\[E \text{ step: } \hat{z}^{(t)}[n] = \hat{x}^{(t)}[n] + h[n] \ast (y[n] - h[n] \ast \hat{x}^{(t)}[n])\]

\[M \text{ step: } \hat{x}^{(t+1)}[n] = \frac{\max\left(\left(\hat{z}^{(t)}[n]\right)^2 - \tau \sigma^2_n\right)}{\hat{z}^{(t)}[n]}\]

\(\hat{x}^{(t)}[n]\) is estimate of signal at nth iteration

\(\hat{z}^{(t)}[n]\) is estimate of missing data
Wiener
Deconvolution and Decomposition
Deconvolution

Given input $y[n]$ which is the received full waveform:

$$y[n] = h[n] \ast x[n] + \eta[n]$$

and

$x[n]$ is the full waveform representation of scene
$h[n]$ is system response (optics, electronics, atmosphere)
$\eta[n]$ is white Gaussian noise

estimate via deconvolution

$x[n]$

which is the signal of interest
Wiener Deconvolution

\[ y[n] = h[n] * x[n] + \eta[n] \]

\[ \text{Wiener Filter} = \frac{H^*}{H^* \cdot H + \alpha \cdot \text{NSR}} \]

\[ \text{NSR} \equiv \text{noise to signal ratio} \]
Decomposition
Sensor Coordinates to Georeferenced Coordinates

\[ \langle X, Y, Z \rangle_L \quad \text{TO} \quad \langle \text{LAT}, \text{LON}, \text{ALT} \rangle_{\text{WGS84}} \]
LiDAR data: Georeferencing

<lat, lon, height>

NSF LiDAR data
LiDAR data: Georeferencing

$r_L$

<lat, lon, height>

NSF data
Quadcopter and Geometry

$M = \text{Local Level Map Coordinates}$
Object in LiDAR Sensor Coordinates

LiDAR Mapping Equation

\[ \mathbf{r}_{o,M} = \mathbf{r}_{\text{INS},M} + \mathbf{R}^M_{\text{INS}} (\mathbf{R}^{\text{INS}}_L \cdot \mathbf{r}_L + \mathbf{b}_{L,\text{INS}}) \]
Sensor: Spherical to Cartesian

\[ r_L = \langle r, \theta, \varphi \rangle_L = \langle x, y, z \rangle_L \]

\[ x = r \sin \theta \cos \varphi \]
\[ y = r \sin \theta \sin \varphi \]
\[ z = r \cos \theta \]

Source: Velodyne Inc.
Object in LiDAR Sensor Coordinates

LiDAR Mapping Equation

\[ \mathbf{r}_{o,M} = \mathbf{r}_{INS,M} + M_R_{INS}(^{INS}R_L \cdot \mathbf{r}_L + \mathbf{b}_{L,INS}) \]
Local Level (Map) and ECEF

Earth Centered Earth Fixed (ECEF)

Local Level (MAP) (ENU)
Local Level (Map) to ECEF

\[ \mathbf{r}_{o,ECEF} = \mathcal{ECEF} f_M (\mathbf{r}_{o,M}, \mathbf{r}_{\text{origin},M}) \]

Diagram: Wang, Huynh, Williamson
ECEF to Geodetic (Lat, Lon, Alt)

Diagram: Wang, Huynh, Williamson
LiDAR data: Georeferenced

<34.5 N, 115W, 245 m>
LiDAR Range Extraction and Georeferencing

- Utilizes Estimation and Detection theory
  - Methods of optimal detection
- Wonderful application of
  - Statistics, probability, and linear algebra
- Involves Geodesy
  - Mappings between world referenced coordinate systems
References

1) *Airborne Topographic Lidar Manual*, Michael Renslowl, Editor

2) Elements of Photogrammetry, Wolf and DeWitt


4) Empirical Comparison of Full-Waveform Lidar Algorithms: Range Extraction and Discrimination Performance; Photogrammetric Engineering and Remote Sensing; Christopher E. Parrish, Inseong Jeong, Robert Nowak, and R. Smith; August 2011


6) 3D vegetation mapping using small-footprint full-waveform airborne laser scanners. Available from: https://www.researchgate.net/248978086_fig8_Figure-2-Gaussian-decomposition-of-the-LMS-Q560-waveform-showing-the-recorded-waveform [accessed 19 Mar, 2017]


