

BadgerWorks

Topic: Collaborative Visual Area Coverage

Dr. Kostas Alexis (CSE)

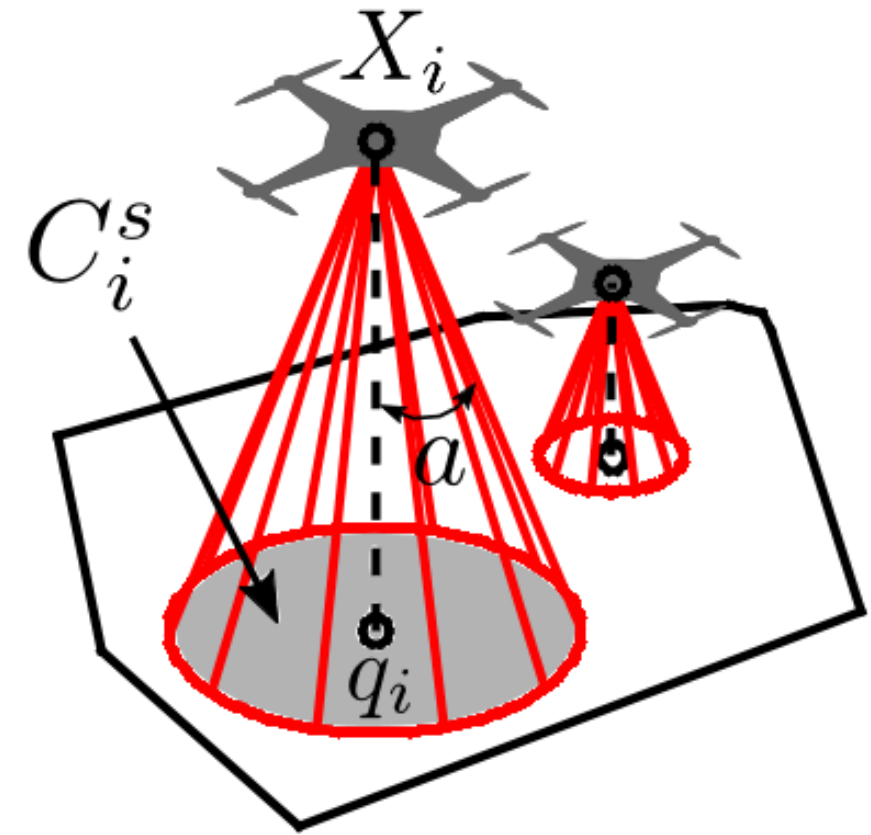
Slides primarily based on “Collaborative Visual Area Coverage using Unmanned Aerial Vehicles”, S. Papatheodorou, A. Tzes, Y. Stergiopoulos

Distributed Multi-Robot Coverage

Sung Lee, Yancy Diaz-Mercado, and Magnus Egerstedt
GRITS Lab, Georgia Tech
March 2014

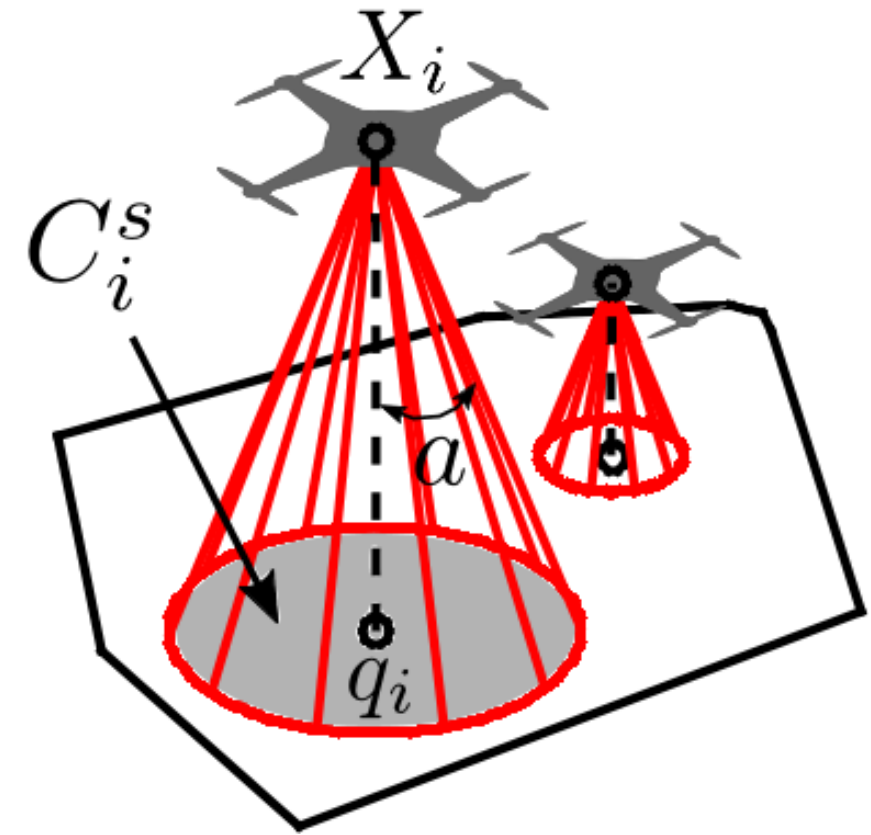
Collaborative Visual Area Coverage

- Visual Area Coverage using a team of Aerial Robots.
 - 2D Plane to be covered.
 - Nadir-looking camera



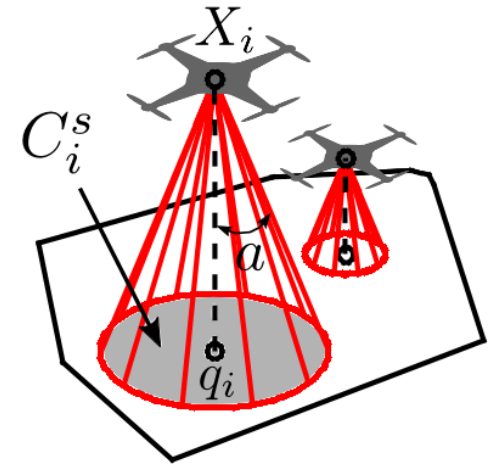
Collaborative Visual Area Coverage

- ▶ Visual Area Coverage using a team of Aerial Robots.
 - ▶ Distributed control law.
 - ▶ Account for the quality of observation.



Problem Statement

- Environment
- Aerial Robots Team
- Sensing Performance



Problem Statement

- Environment
 - $\Omega \subset \mathbb{R}^2$ be a compact convex region under surveillance.
- Aerial Robots Team
- Sensing Performance

- **Environment**

- $\Omega \subset \mathbb{R}^2$ be a compact convex region under surveillance.

- Aerial Robots Team

- Sensing Performance

Problem Statement

- Environment

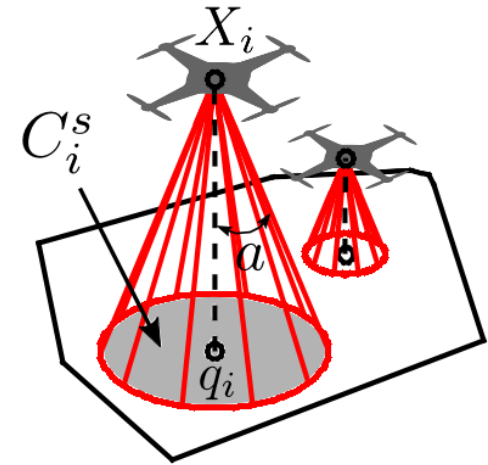
- $\Omega \subset \mathbb{R}^2$ be a compact convex region under surveillance.

- Aerial Robots Team

- $X_i = [x_i, y_i, z_i]^T, i \in I_n, I_n = \{1, \dots, n\}$ representing the position of each robot.
 - $q_i = [x_i, y_i]^T, q_i \in \Omega$ representing the projection to surveillance space.
 - $z_i \in [z_i^{\min}, z_i^{\max}]$ defining minima and maxima for the altitude of every robot
 - Kinodynamic Model ($[u_{i,q}, u_{i,z}]$ the control input of thrust/direction for each robot)

$$\begin{aligned}\dot{q}_i &= u_{i,q}, \quad q_i \in \Omega, \quad u_{i,q} \in \mathbb{R}^2, \\ \dot{z}_i &= u_{i,z}, \quad z_i \in [z_i^{\min}, z_i^{\max}], \quad u_{i,z} \in \mathbb{R}.\end{aligned}$$

- Sensing Performance



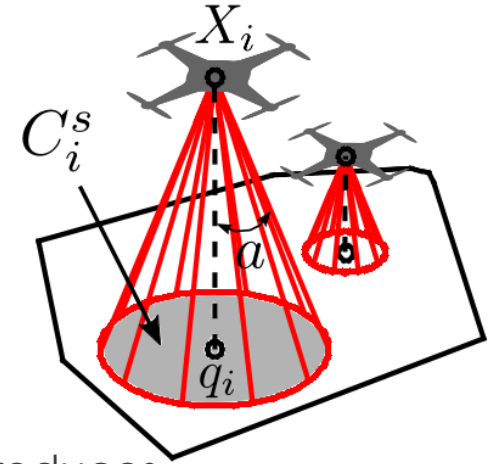
Problem Statement

► Coverage Quality

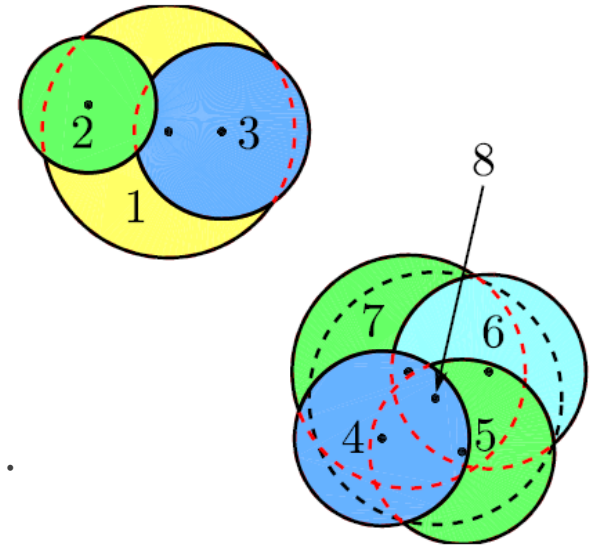
- $f_i(q): \mathbb{R}^2 \rightarrow [0,1]$ coverage quality function that depends on X_i
- It can be considered that as the altitude increases, the quality reduces.
- For each point $q \in \Omega$ an importance weight is assigned via the space density function $\phi: \Omega \rightarrow \mathbb{R}^+$ encapsulating any a priori information regarding the region of interest.
- Therefore, the Coverage Quality objective is:

$$\mathcal{H} \triangleq \int_{\Omega} \max_{i \in I_n} f_i(q) \phi(q) dq.$$

- For simplicity, it is now assumed that $\phi(q) = 1, \forall q \in \Omega$



Sensed Space Partitioning

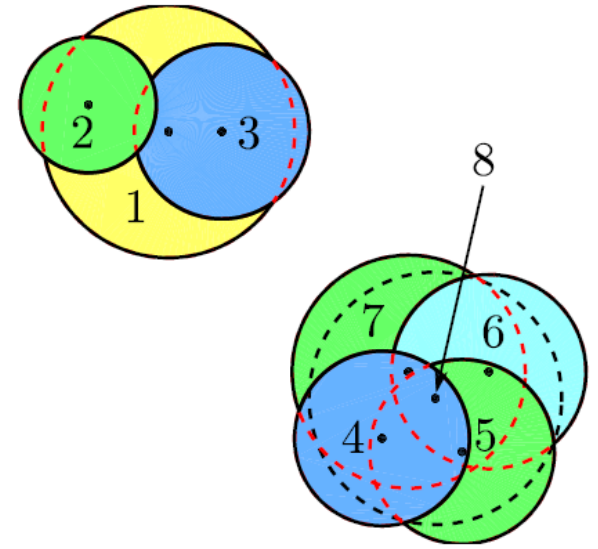


- Only the subset of Ω sensed by the nodes is partitioned.
- Each node is assigned to a cell:

$$W_i \triangleq \{q \in \Omega: f_i(q) \geq f_j(q), j \neq i\}$$

- with the equality holding only at the boundary ∂W_i so that the cells W_i comprise a complete tessellation of the sensed region.

Sensed Space Partitioning

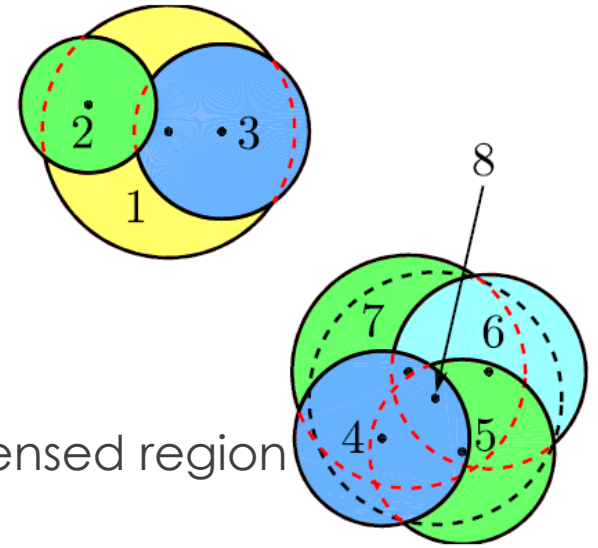


- Neighbors N_i of node i

$$N_i \triangleq \{j \neq i: C_j^s \cap C_i^s \neq \emptyset\}$$

- The neighbors of node i are those nodes that sense at least a part of the region that node i senses.
 - Only the nodes in N_i need to be considered when creating W_i

Sensed Space Partitioning



- ▶ The aforementioned is a complete tessellation of the sensed region

$$\bigcup_{i \in I_n} C_i^s$$

- ▶ But not necessarily a complete tessellation of Ω . We denote the residual space:

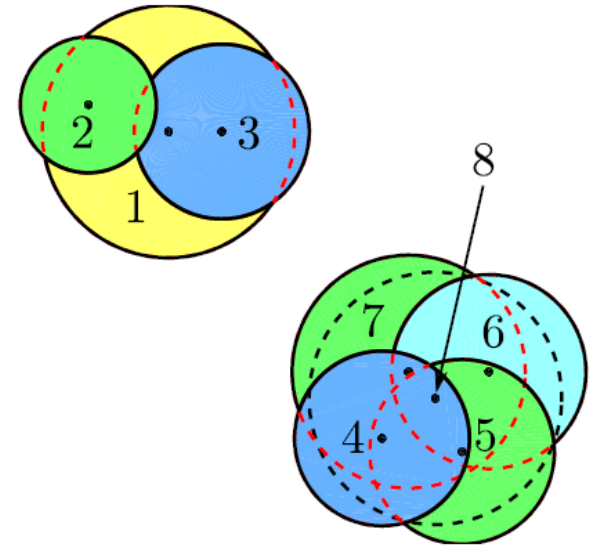
$$\mathcal{O} = \Omega \setminus \bigcup_{i \in I_n} W_i$$

- ▶ Furthermore, it is noted that the resulting cells W_i are compact but not necessarily convex.
 - ▶ It is also possible that a cell W_i consists of multiple disjoint regions (e.g. cell 1)

Sensed Space Partitioning

- Network's Coverage Performance

$$\mathcal{H} = \sum_{i \in I_n} \int_{W_i} f_i(q) \phi(q) dq$$



Distributed Coordination Algorithm

- Given:
 - Kinodynamics model
 - Sensing performance
 - Coverage criterion
- A gradient based control law is designed and utilizes the partitioning of space and ensures monotonous increase of the covered area.

$$\begin{aligned}
 u_{i,q} &= \alpha_{i,q} \left[\int_{\partial W_i \cap \partial \mathcal{O}} n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq + \right. \\
 &\quad \left. \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} v_i^i n_i (f_i(q) - f_j(q)) dq \right] \\
 u_{i,z} &= \alpha_{i,z} \left[\int_{\partial W_i \cap \partial \mathcal{O}} \tan(a) f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial z_i} dq + \right. \\
 &\quad \left. \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} v_i^i \cdot n_i (f_i(q) - f_j(q)) dq \right]
 \end{aligned}$$

Distributed Coordination Algorithm

- ▶ A gradient based control law is designed and utilizes the partitioning of space and ensures monotonous increase of the covered area.

$$\begin{aligned}
 u_{i,q} &= \alpha_{i,q} \left[\int_{\partial W_i \cap \partial \mathcal{O}} n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq + \right. \\
 &\quad \left. \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} v_i^i n_i (f_i(q) - f_j(q)) dq \right] \\
 u_{i,z} &= \alpha_{i,z} \left[\int_{\partial W_i \cap \partial \mathcal{O}} \tan(a) f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial z_i} dq + \right. \\
 &\quad \left. \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} v_i^i \cdot n_i (f_i(q) - f_j(q)) dq \right]
 \end{aligned}$$

- ▶ $\alpha_{i,q}, \alpha_{i,z}$ are positive constants and n_i the outward pointing normal vector of W_i
- ▶ This control law maximizes the performance criterion monotonically

Distributed Coordination Algorithm

- A gradient based control law is designed and utilizes the partitioning of space and ensures monotonous increase of the covered area.

$$u_{i,q} = \alpha_{i,q} \left[\int_{\partial W_i \cap \partial \mathcal{O}} n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq + \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} v_i^i n_i (f_i(q) - f_j(q)) dq \right]$$

$$u_{i,z} = \alpha_{i,z} \left[\int_{\partial W_i \cap \partial \mathcal{O}} \tan(a) f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial z_i} dq + \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} v_i^i \cdot n_i (f_i(q) - f_j(q)) dq \right]$$

$$v_i^i(q) = \begin{bmatrix} \frac{\partial x}{\partial x_i} & \frac{\partial x}{\partial y_i} \\ \frac{\partial y}{\partial x_i} & \frac{\partial y}{\partial y_i} \end{bmatrix} = \mathbb{I}_2$$

$$n_i = \begin{bmatrix} \cos(k) \\ \sin(k) \end{bmatrix}, \quad k \in [0, 2\pi)$$

$$v_i^i(q) = \begin{bmatrix} \frac{\partial x}{\partial z_i} \\ \frac{\partial y}{\partial z_i} \end{bmatrix} = \begin{bmatrix} \tan(a) \cos(k) \\ \tan(a) \sin(k) \end{bmatrix}$$

- $\alpha_{i,q}, \alpha_{i,z}$ are positive constants and n_i the outward pointing normal vector of W_i
- This control law maximizes the performance criterion monotonically

Distributed Coordination Algorithm

► Proof:

► Evaluate time derivative of optimization criterion \mathcal{H}

$$\frac{d\mathcal{H}}{dt} = \sum_{i \in I_n} \left[\frac{\partial \mathcal{H}}{\partial q_i} \dot{q}_i + \frac{\partial \mathcal{H}}{\partial z_i} \dot{z}_i \right]$$

► The usage of a gradient based control law in the form

$$u_{i,q} = \alpha_{i,q} \frac{\partial \mathcal{H}}{\partial q_i}, \quad u_{i,z} = \alpha_{i,z} \frac{\partial \mathcal{H}}{\partial z_i}$$

► Will result in a monotonous increase in \mathcal{H}

Distributed Coordination Algorithm

► Proof:

► Applying Leibnitz integral rule

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial q_i} &= \sum_{i \in I_n} \left[\int_{\partial W_i} v_i^i n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq \right] \\ &= \int_{\partial W_i} v_i^i n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq + \\ &\quad \sum_{j \neq i} \left[\int_{\partial W_j} v_j^i n_j f_j(q) dq + \int_{W_j} \frac{\partial f_j(q)}{\partial q_i} dq \right]\end{aligned}$$

$$v_j^i(q) \triangleq \frac{\partial q}{\partial q_i}, \quad q \in \partial W_j, \quad i, j \in I_n$$

Distributed Coordination Algorithm

► Proof:

► Since:

$$\frac{\partial f_j(q)}{\partial q_i} = 0$$

► we obtain:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial q_i} = & \int_{\partial W_i} v_i^i n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq + \\ & \sum_{j \neq i} \int_{\partial W_j} v_j^i n_j f_j(q) dq \end{aligned}$$

- The three terms indicate how a movement of node i affects the boundary of its cell and the boundaries of the cells of other nodes.
- Only the cells W_j which have a common boundary with W_i will be affected therefore giving rise to a distributed control law.

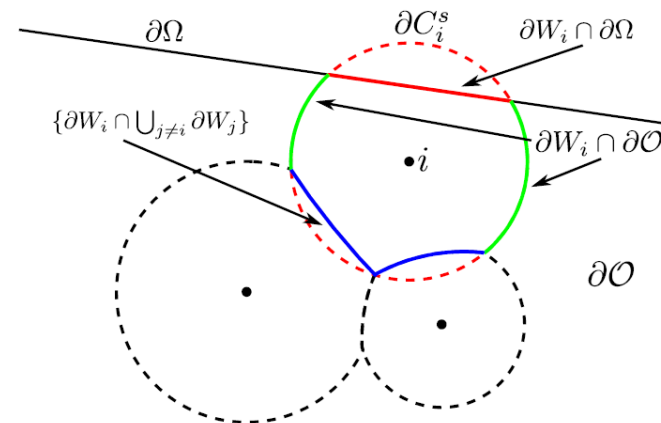
Distributed Coordination Algorithm

► Proof:

- The boundary ∂W_i can be decomposed in disjoint sets as

$$\partial W_i = \{\partial W_i \cap \partial \Omega\} \cup \{\partial W_i \cap \partial \mathcal{O}\} \cup \left\{ \bigcup_{j \neq i} (\partial W_i \cap \partial W_j) \right\}$$

- These sets represent the parts of ∂W_i that lie on the boundary of Ω , the boundary of the node's sensing region and the parts that are common between the boundary of the cell of node i and those of other nodes.



∂W_i decomposition into disjoint sets.

Distributed Coordination Algorithm

► Proof:

- At $q \in \partial\Omega$ it holds that $v_i^i = 0_{2 \times 2}$ since we assume the region of interest to be static.
- Additionally, since only the common boundary $\partial W_j \cap \partial W_i$ of node i with another node j is affected by the movement of node i , $\frac{\partial \mathcal{H}}{\partial q_i}$ can be simplified as

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial q_i} = & \int_{\partial W_i \cap \partial \mathcal{O}} v_i^i n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq + \\ & \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} v_i^i n_i f_i(q) dq + \\ & \sum_{j \neq i} \int_{\partial W_j \cap \partial W_i} v_j^i n_j f_j(q) dq. \end{aligned}$$

Distributed Coordination Algorithm

► Proof:

- Because the boundary $\partial W_i \cap \partial W_j$ is common among nodes i and j , it holds true that $v_j^i = v_i^i$ when evaluated over it and that $n_j = -n_i$.
- Finally, the sums and the integrals within then can be combined, producing the final form of the planar control law

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial q_i} = & \int_{\partial W_i \cap \partial \mathcal{O}} n_i f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial q_i} dq + \\ & \sum_{j \neq i} \int_{\partial W_j \cap \partial W_i} v_i^i n_i (f_i(q) - f_j(q)) dq. \end{aligned}$$

- Similarly, by using the same ∂W_i decomposition and defining $v_j^i(q) \triangleq \frac{\partial q}{\partial z_i}$, $q \in \partial W_j$, $i, j \in I_n$, the altitude control is:

Distributed Coordination Algorithm

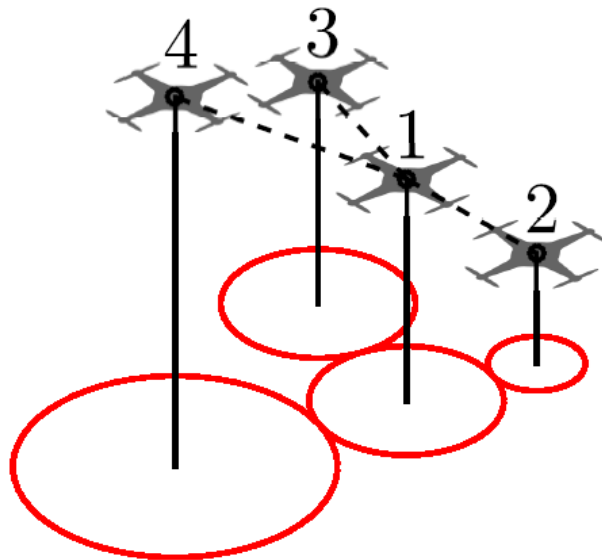
► Proof:

► Similarly, by using the same ∂W_i decomposition and defining $v_j^i(q) \triangleq \frac{\partial q}{\partial z_i}$, $q \in \partial W_j$, $i, j \in I_n$, the altitude control is:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial z_i} = & \int_{\partial W_i \cap \partial \mathcal{O}} \tan(a) f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial z_i} dq + \\ & \sum_{j \neq i} \int_{\partial W_j \cap \partial W_i} v_j^i \cdot n_i (f_i(q) - f_j(q)) dq \end{aligned}$$

Distributed Coordination Algorithm

- ▶ The cell W_i of node i is affected by its neighbors N_i therefore resulting in a distributed control law.
- ▶ The finding of the neighbors N_i depends on their coordinates $X_j, j \in N_i$.
- ▶ The computation of the N_i set demands node i to be able to communicate with all nodes within a sphere centered around X_i and radius r_i^c



$$r_i^c = \max \left\{ 2z_i \tan a, \left(z_i + z^{\min} \right)^2 \tan^2 a + \left(z - z^{\min} \right)^2, \right. \\ \left. \left(z_i + z^{\max} \right)^2 \tan^2 a + \left(z - z^{\max} \right)^2 \right\}$$

Distributed Coordination Algorithm

- Stable Altitude:

- The altitude control law $u_{i,z}$ moves each robot towards an altitude in which $u_{i,z} = 0$ which corresponds to an equilibrium point for that particular node.
- This is called the “stable altitude? And it is the solution wrt z_i of the equation:

$$u_{i,z} = 0 \Rightarrow$$
$$\int_{\partial W_i \cap \partial \mathcal{O}} \tan(a) f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial z_i} dq +$$
$$\sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} \mathbf{v}_i^i \cdot \mathbf{n}_i (f_i(q) - f_j(q)) dq = 0.$$

- Both integrals over ∂W_i are non-negative, whereas the integral over W_i is negative since coverage quality decreases as altitude increases.
- The stable altitude is not common among nodes as it depends on one's neighbors N_i and is not constant over time since the neighbors change over time.

Distributed Coordination Algorithm

- Optimal Altitude:

- We call optimal altitude z^{opt} the altitude a node would reach if:
 - It had no neighbors and
 - No part of W_i on $\partial\Omega$.

$$\int_{\partial C_i^s} \tan(a) f_i(q) dq + \int_{W_i} \frac{\partial f_i(q)}{\partial z_i} dq = 0$$

- We denote the sensing region of a node i at z^{opt} as $C_{i,opt}^s$ and the value of the criterion when all nodes are located at z^{opt} as \mathcal{H}_{opt}

Coverage Quality Functions

- ▶ The function $f_i, i \in I_n$ is required to have the following properties:
 - ▶ $f_i(q) = 0, \forall q \notin C_i^S$
 - ▶ $f_i(q) = 0, \forall q \in \partial C_i^S$
 - ▶ $f_i(q)$ is first order differentiable with respect to q_i and z_i , or $\frac{\partial f_i(q)}{\partial q_i}$ and $\frac{\partial f_i(q)}{\partial z_i}$ exist within C_i^S
 - ▶ $f_i(q)$ is symmetric around the z-axis
 - ▶ $f_i(q)$ is a decreasing function of z_i
 - ▶ $f_i(q)$ is a non-increasing function of $\|q - q_i\|$
 - ▶ $f_i(q_i) = 1$ when $z_i = z^{min}$ and $f_i(q_i) = 0$ when $z_i = z^{max}$

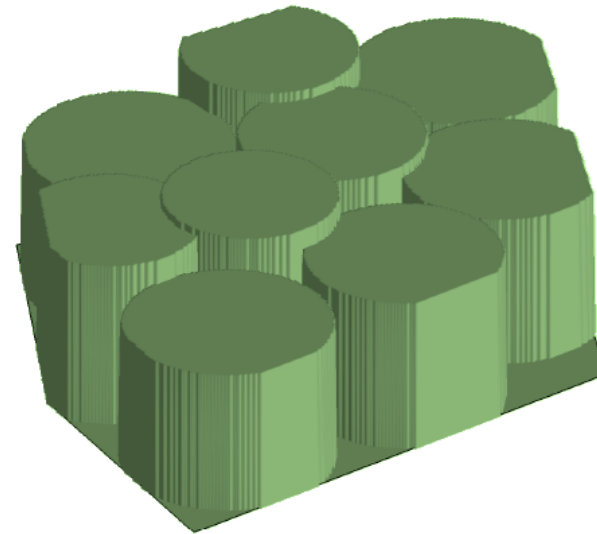
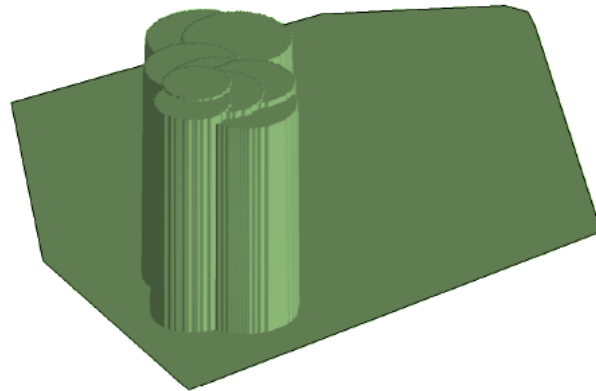
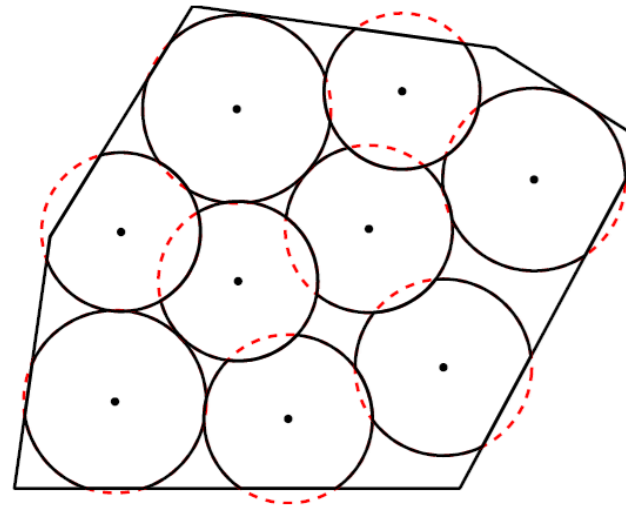
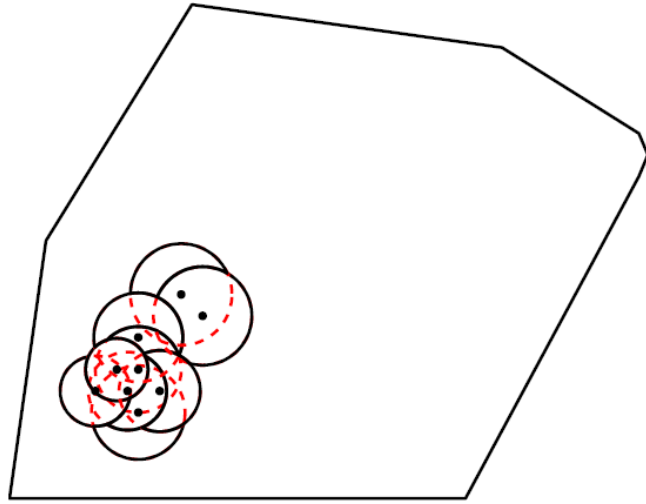
Coverage Quality Functions

- Specific Case: Decreasing Coverage Quality as distance increases
 - Coverage quality of a point $q \in C_i^s$ is maximum directly below the node and decreases as $\|q - q_i\|$ increases.
 - One such function is an inverted paraboloid whose maximum value depends on z_i and could be defined as

$$f_i^p(q) = \begin{cases} \left[1 - \frac{1-b}{[z_i \tan(a)]^2} [(x-x_i)^2 + (y-y_i)^2] \right] f_i^u, & q \in C_i^s \\ 0, & q \notin C_i^s \end{cases}$$

- where $b \in (0,1)$ is the coverage quality on ∂C_i^s as a percentage of the coverage quality on q_i and f_i^u is the uniform coverage quality function defined previously and is used to set the maximum value of the paraboloid.

Simulation Example



Applications in DDD-Robotics

- **Nuclearized Robotics:** Rapid search for illicit nuclear radiation source detection
 - Possibly change the performance function.
 - More difficult problem because no prior knowledge of source location.
 - Can be approached rather exhaustively with a small “coverage” radius.
- **Search and Rescue Robotics:** Rapid surveying of earthquake zone
- **Security Robotics:** Surveillance in remote areas
 - Critical aspect to account for updates of the environment through re-calculating the weights over the plane of coverage.

Definition of Semester-Long Project Goals

➤ **Nuclearized Robotics**

- Task 1: Nuclear Source Localization exploiting Multi-Detector Information and Visual/Depth Cues
- Task 2: RotorS-Nuclearized Robotics

➤ **Multi-Robot Coverage**

- Task 1: Radiation Field Estimation
- Task 2: Multi-Robot Area Coverage in 2D and 3D

➤ **Rapid Remote Access in DVE Environments**

- Task 1: Ground Rover Platform for Remote Access
- Task 2: DVE SLAM onboard a Ground and Aerial Robotic Platform

Find out more

- ▶ Cortes, J., Martinez, S., Karatas, T. and Bullo, F., 2004. Coverage control for mobile sensing networks. IEEE Transactions on robotics and Automation, 20(2), pp.243-255.
- ▶ Bullo, F., Cortes, J. and Martinez, S., 2009. Distributed control of robotic networks: a mathematical approach to motion coordination algorithms. Princeton University Press.
- ▶ Papatheodorou, S. and Tzes, A., 2017, December. Cooperative visual convex area coverage using a tessellation-free strategy. In Decision and Control (CDC), 2017 IEEE 56th Annual Conference on (pp. 4662-4667). IEEE.
- ▶ McNew, J.M., Klavins, E. and Egerstedt, M., 2007, April. Solving coverage problems with embedded graph grammars. In International Workshop on Hybrid Systems: Computation and Control (pp. 413-427). Springer, Berlin, Heidelberg.
- ▶ Schwager, M., Rus, D. and Slotine, J.J., 2009. Decentralized, adaptive coverage control for networked robots. The International Journal of Robotics Research, 28(3), pp.357-375.



A black and white photograph of a drone flying in front of a construction site. The drone is in the foreground, slightly out of focus, with its four rotors visible. In the background, several large construction cranes are visible, also out of focus, against a bright sky. The overall scene is a construction site.

Thank you!

Please ask your question!