CS302 - Data Structures *using C++*

Topic: Balanced Search Trees - Introduction

Kostas Alexis

2-3 Trees

- General Search Tree
- Interior nodes are either 2-nodes or 3-nodes
  - 2-node has one data item and two children
  - 3-node has two data items and three children
    - Simple implementations may use 3-nodes for both

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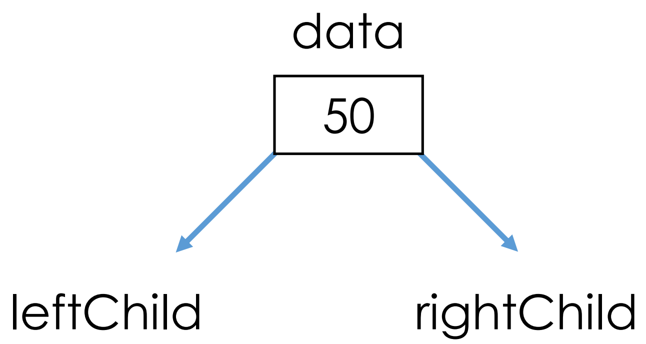

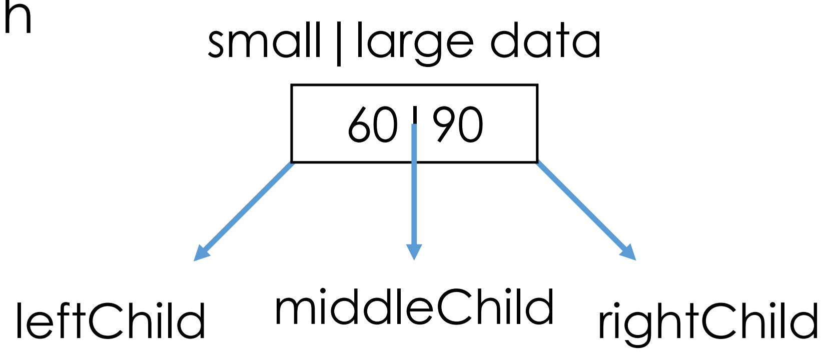

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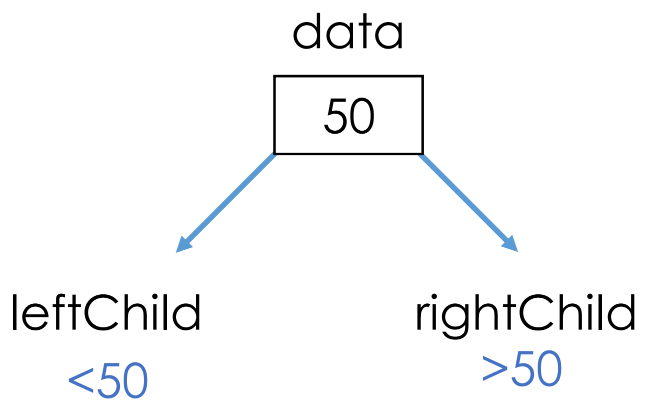

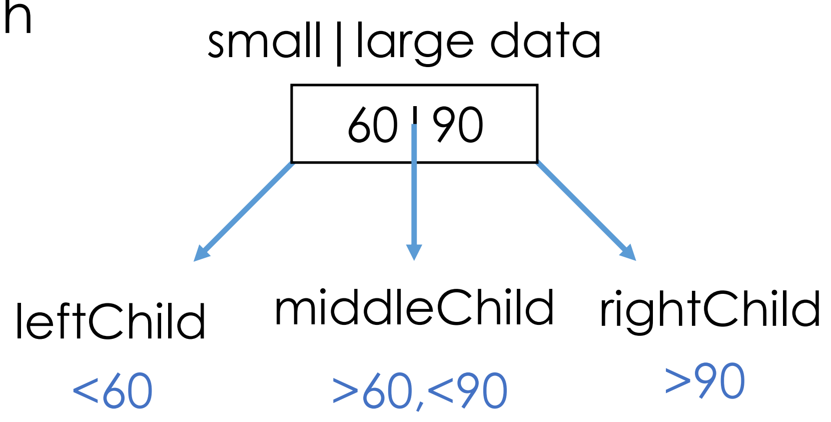

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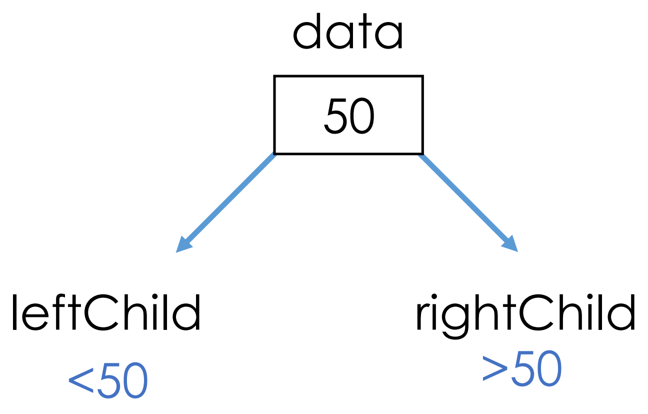
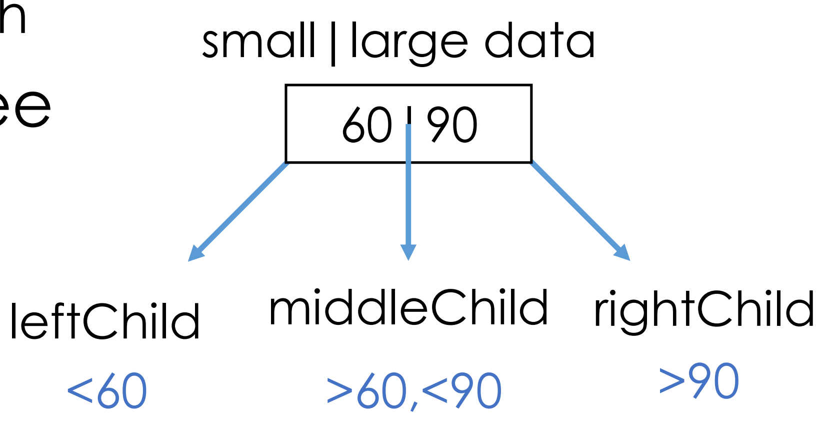
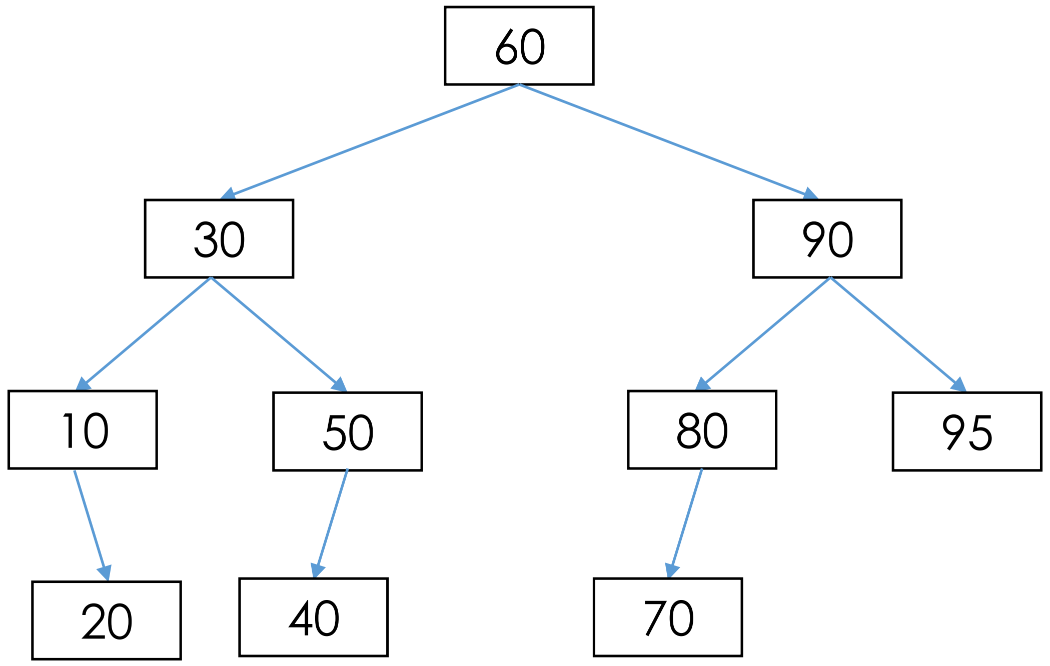
- Are never taller than minimum-height binary tree
  - A 2-3 tree with n nodes never has height greater than $\log_2(n+1)$



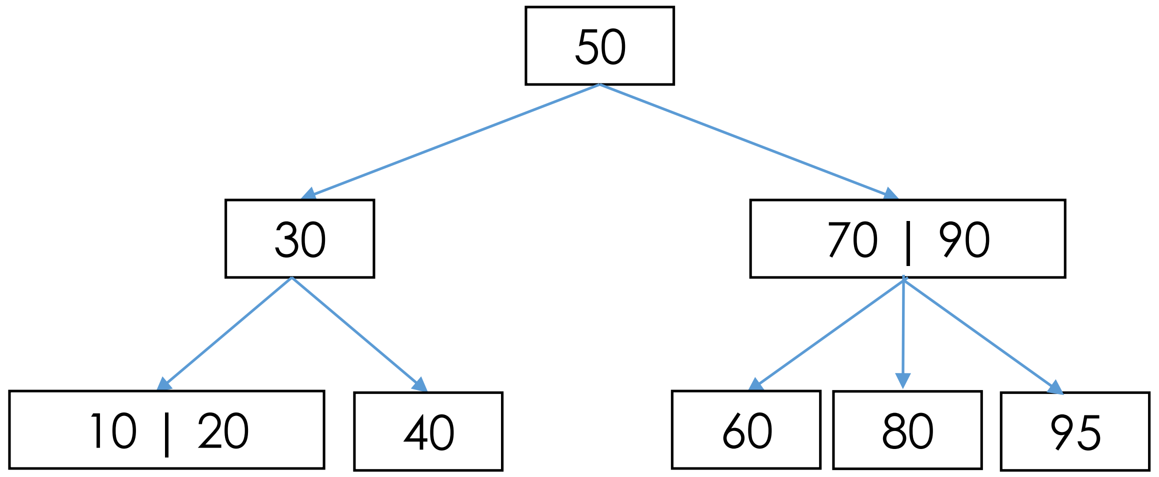


2-3 Trees

A Balanced Binary Search Tree

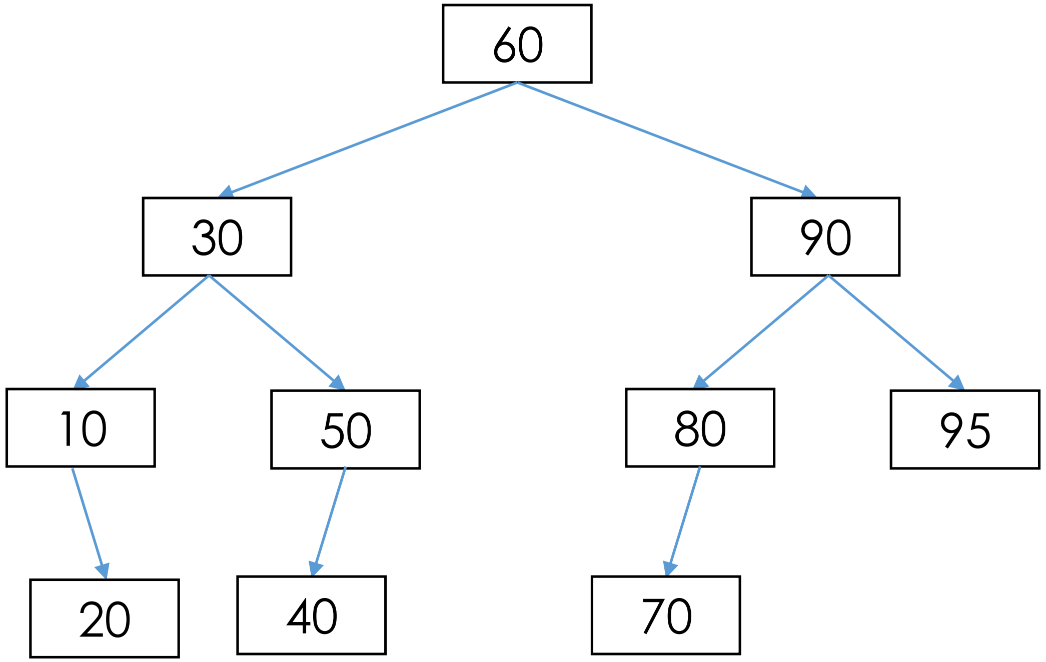


2-3 Tree with the same elements

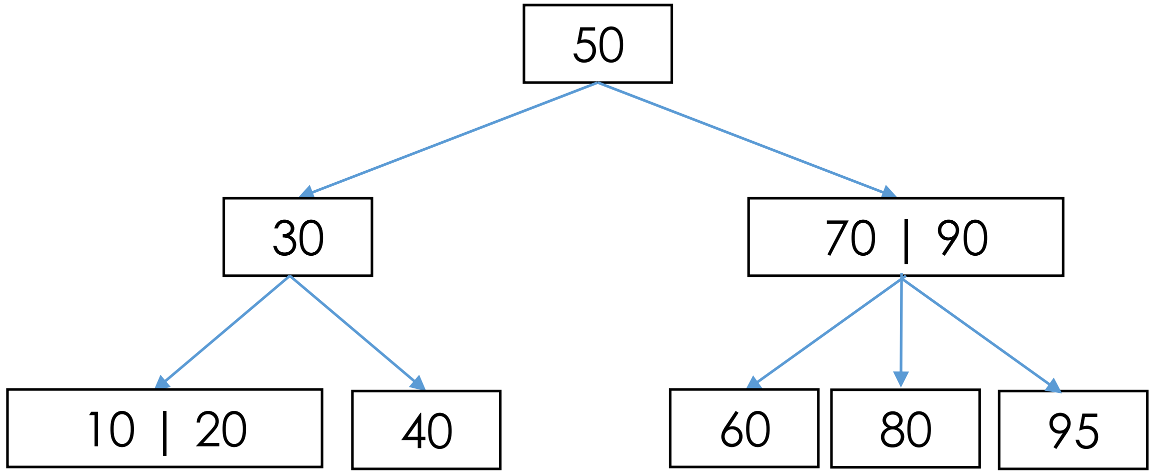


2-3 Trees

A Balanced Binary Search Tree – Height = 4



2-3 Tree with the same elements – Height = 3



Traversing 2-3 Trees

- To traverse a 2-3 Tree
  - Perform the analogue of an in-order traversal
    - Leftmost subtree,
    - Left value,
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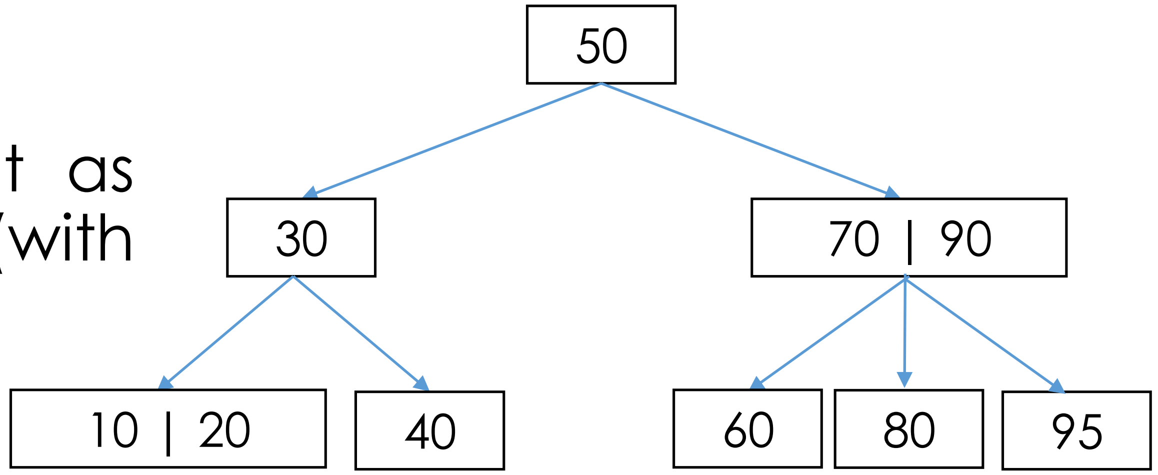
- Searching a 2-3 tree is as efficient as searching the shorted binary tree (with the same values)
  - $O(\log_2 n)$



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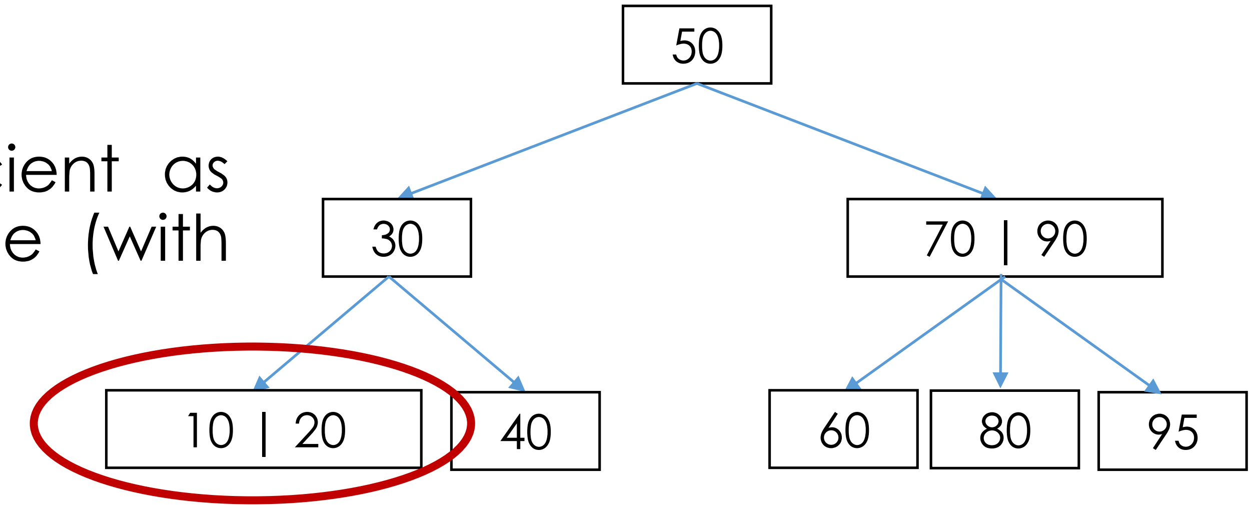

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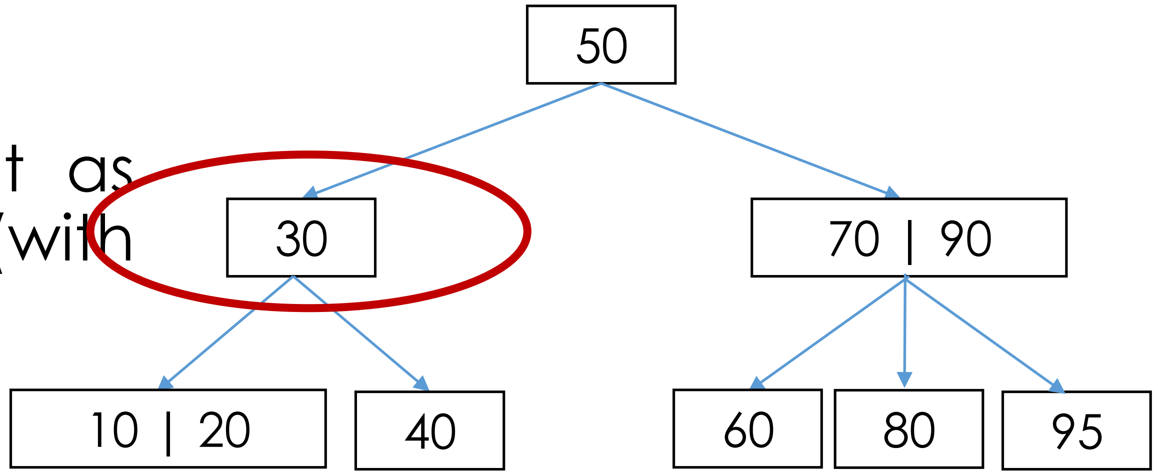

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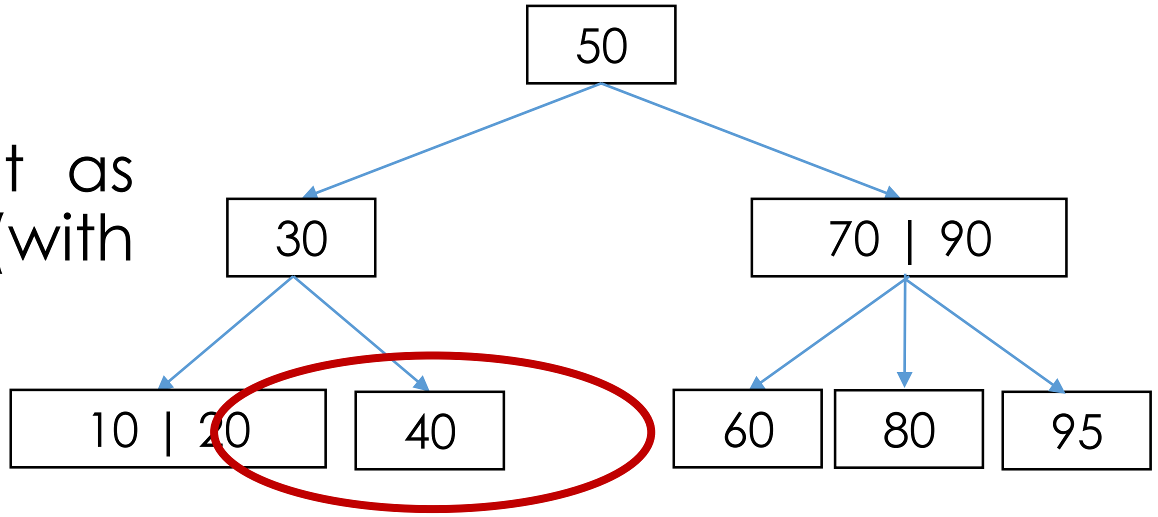

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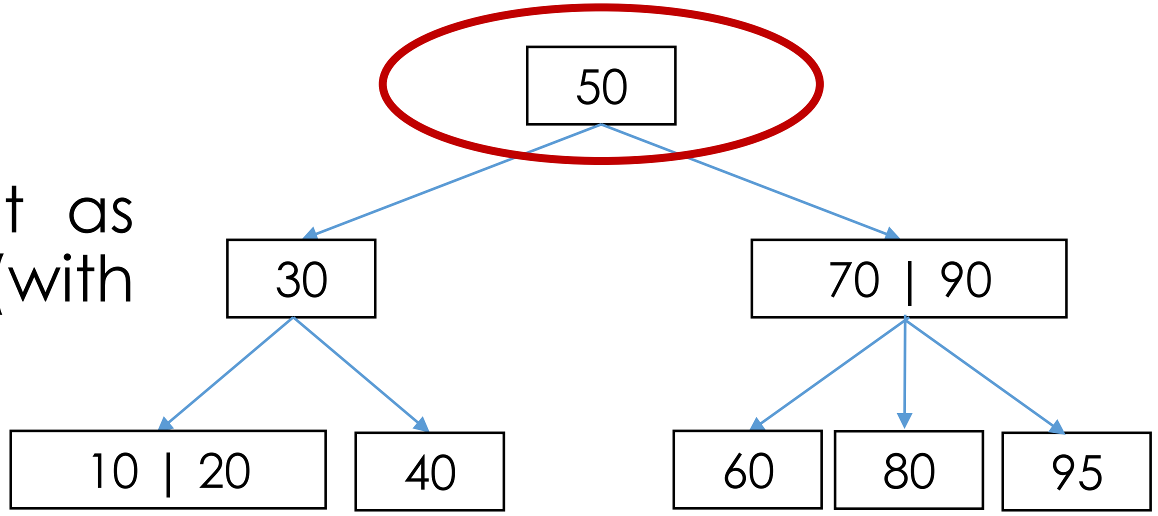

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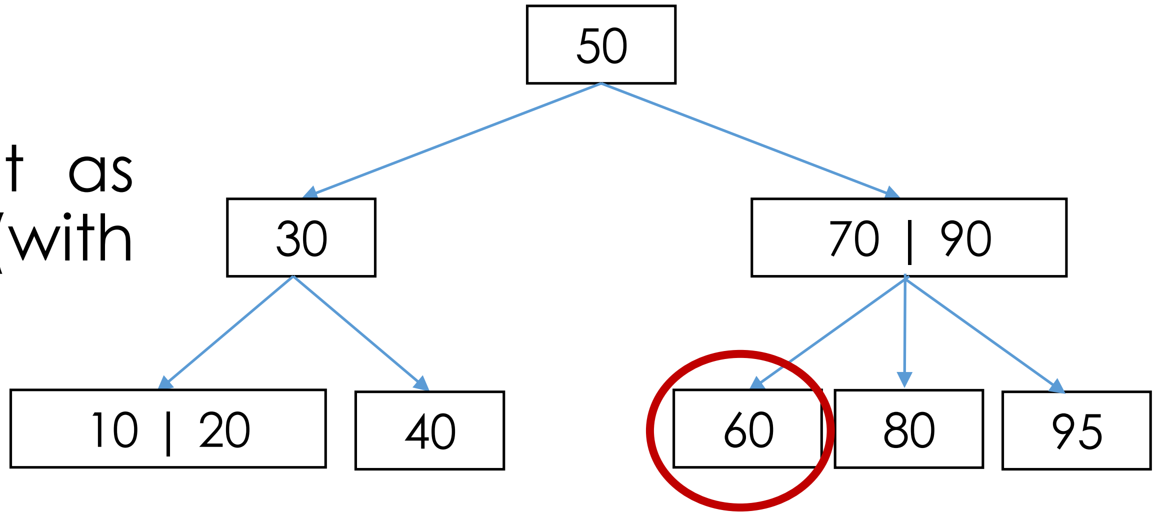

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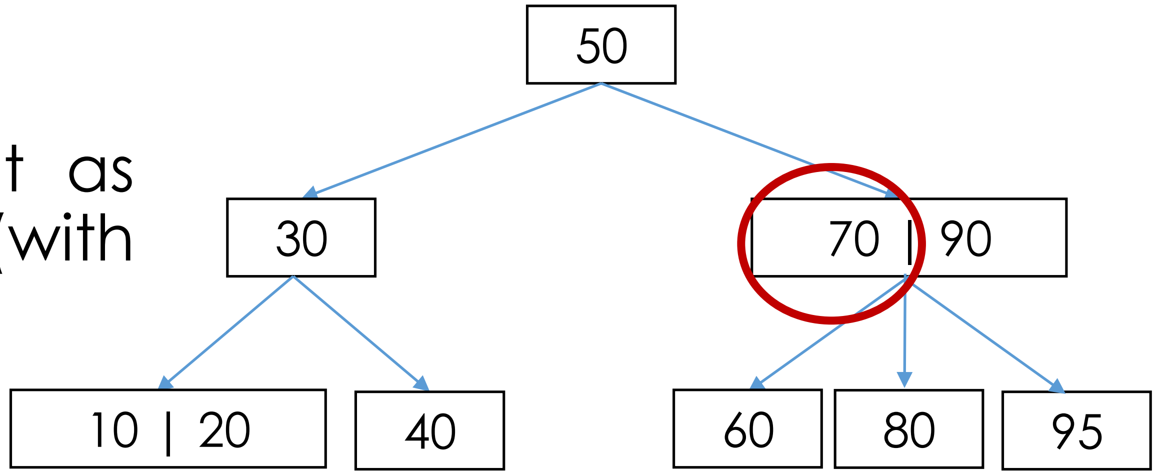

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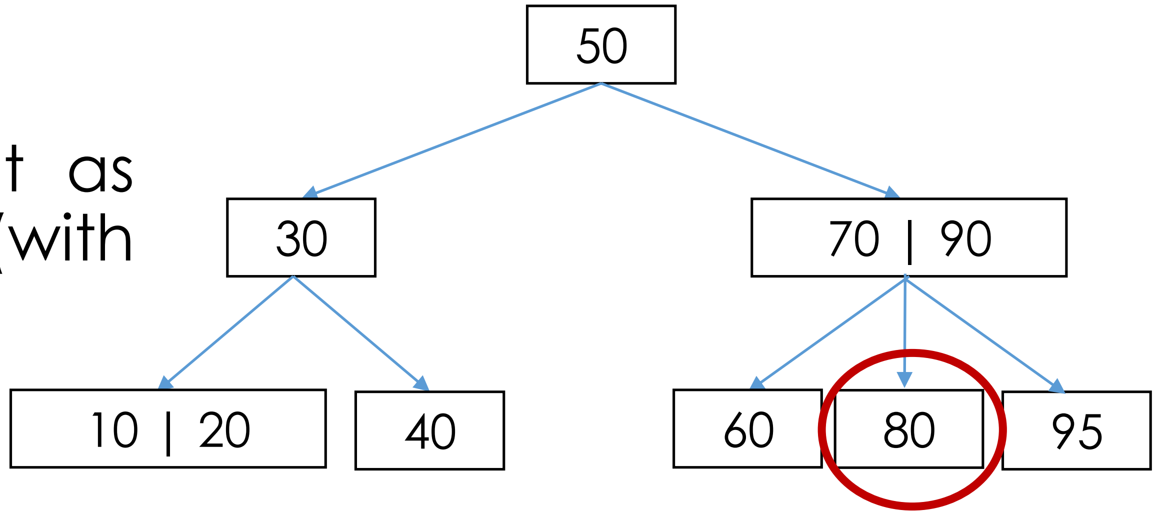

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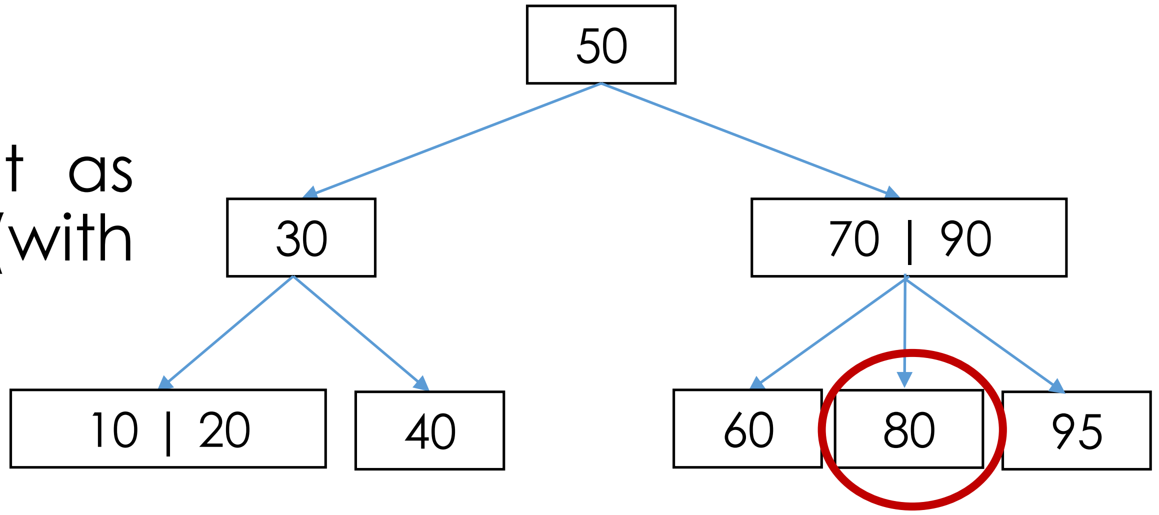

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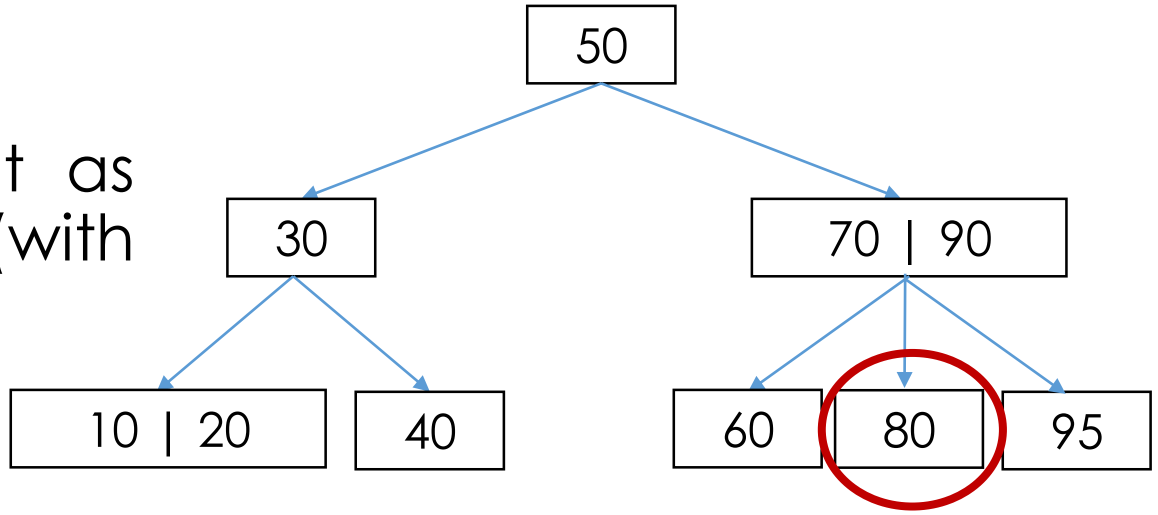

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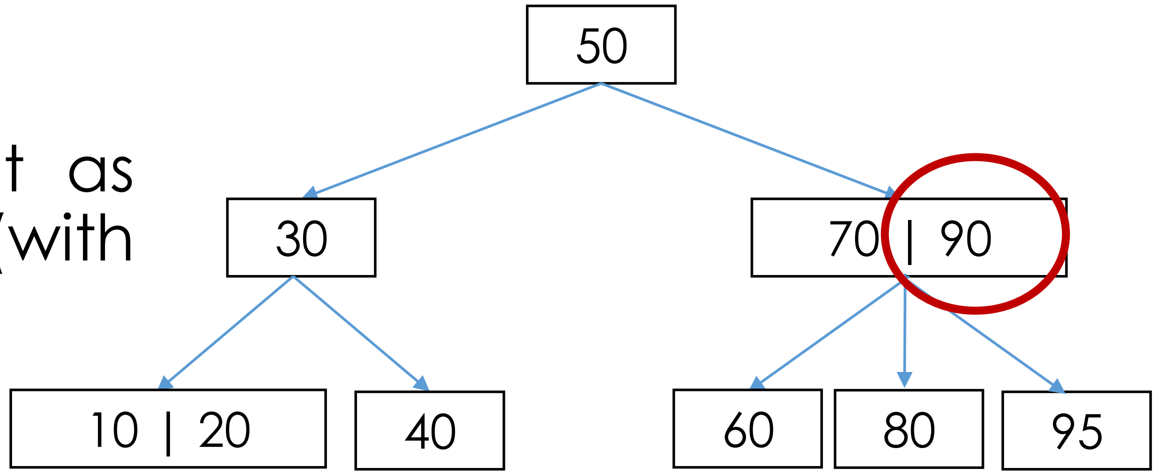

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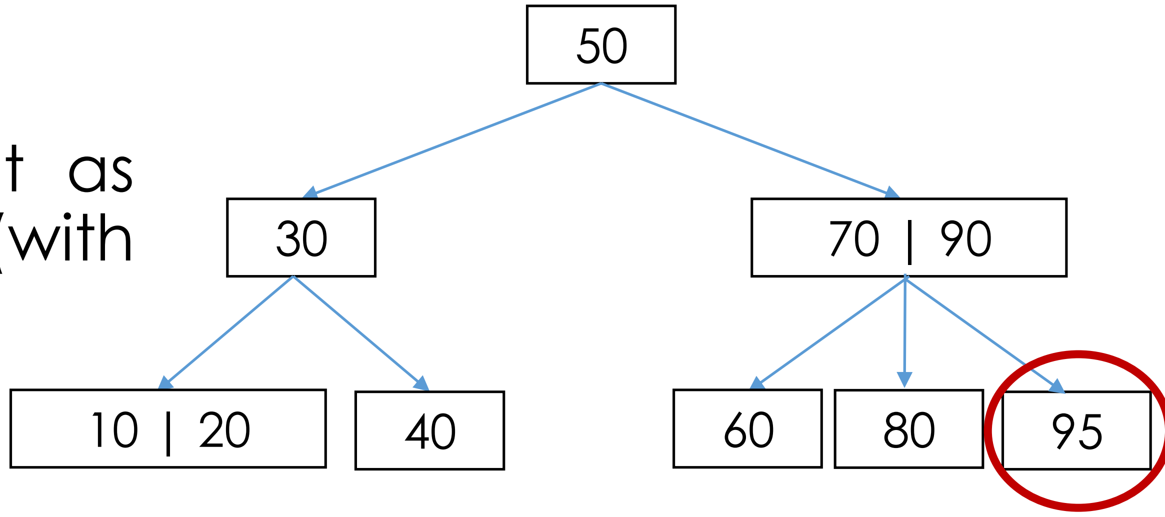

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Adding to 2-3 Trees

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    - Result is a subtree of 2-nodes

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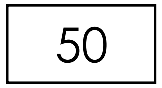

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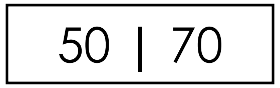

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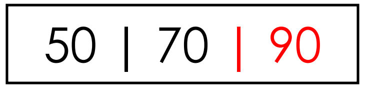

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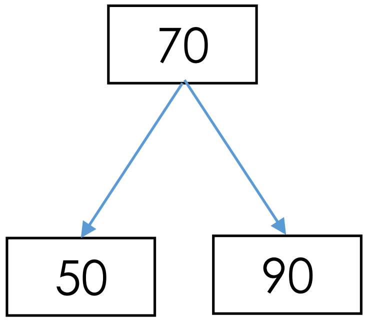

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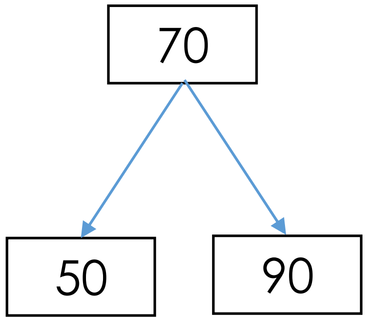

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    - Result is a subtree of 2-nodes
- To insert an item with key k into a 2-3 tree
  - Locate the leaf at which the search for k would terminate
  - Insert the new item k into the leaf
  - If the leaf now contains only two items – done
  - If the leaf now contains three items, split the leaf into two nodes and move the middle value into parent

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- When an internal node contains three items
  - Split the node into two nodes
  - Accommodate the node’s children
- When the root contains three items
  - Split the root into two nodes
  - Create a new root node
- Tree grows in height

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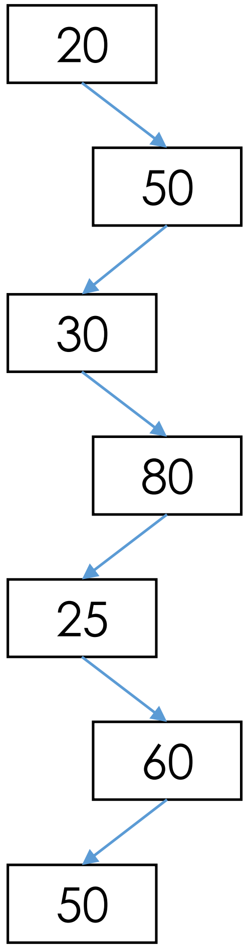

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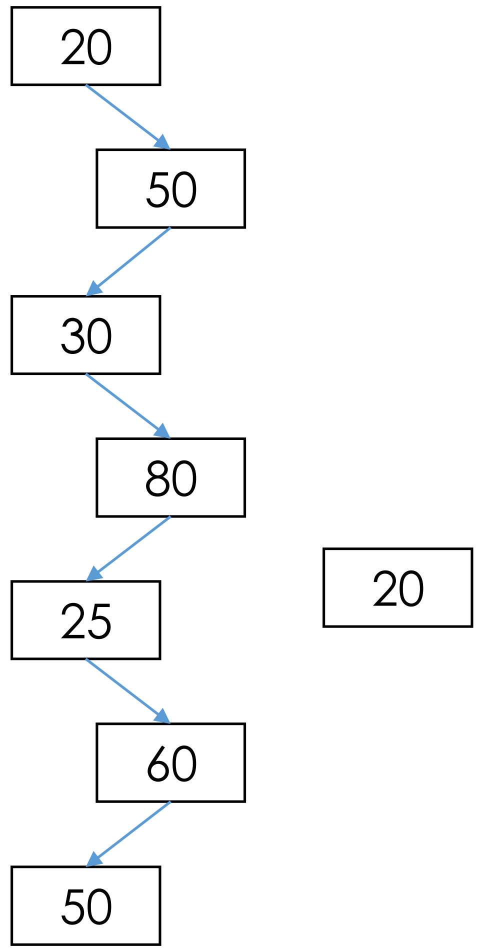

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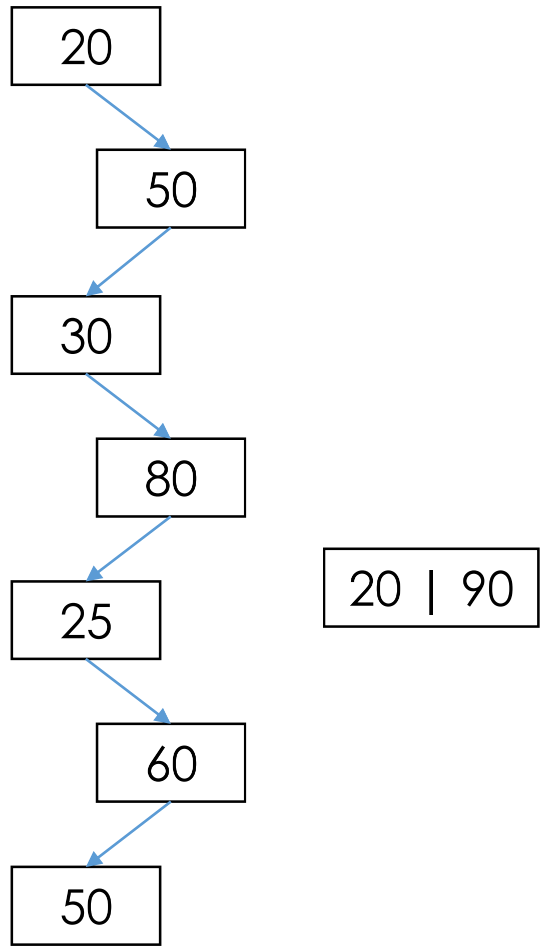

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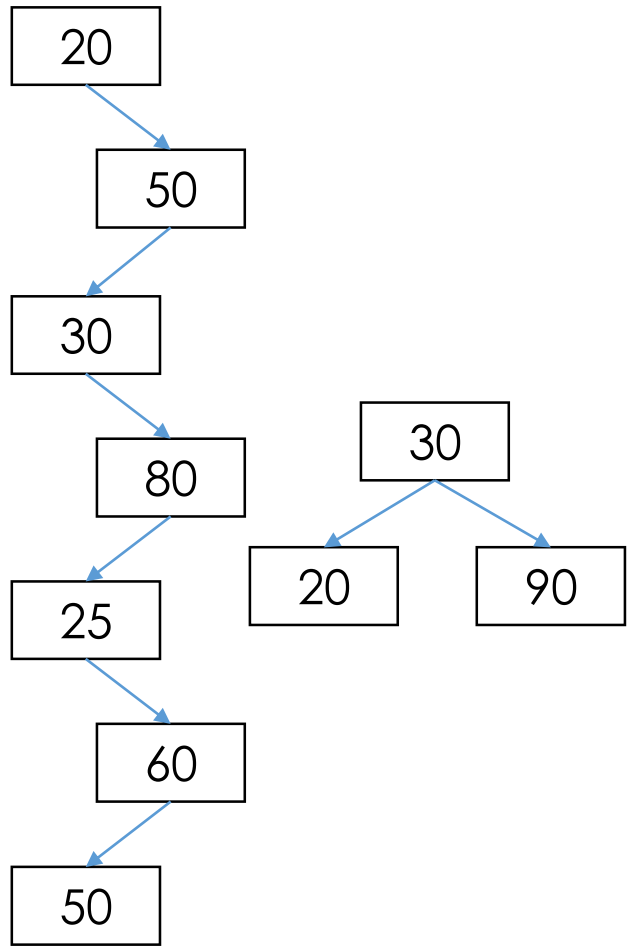

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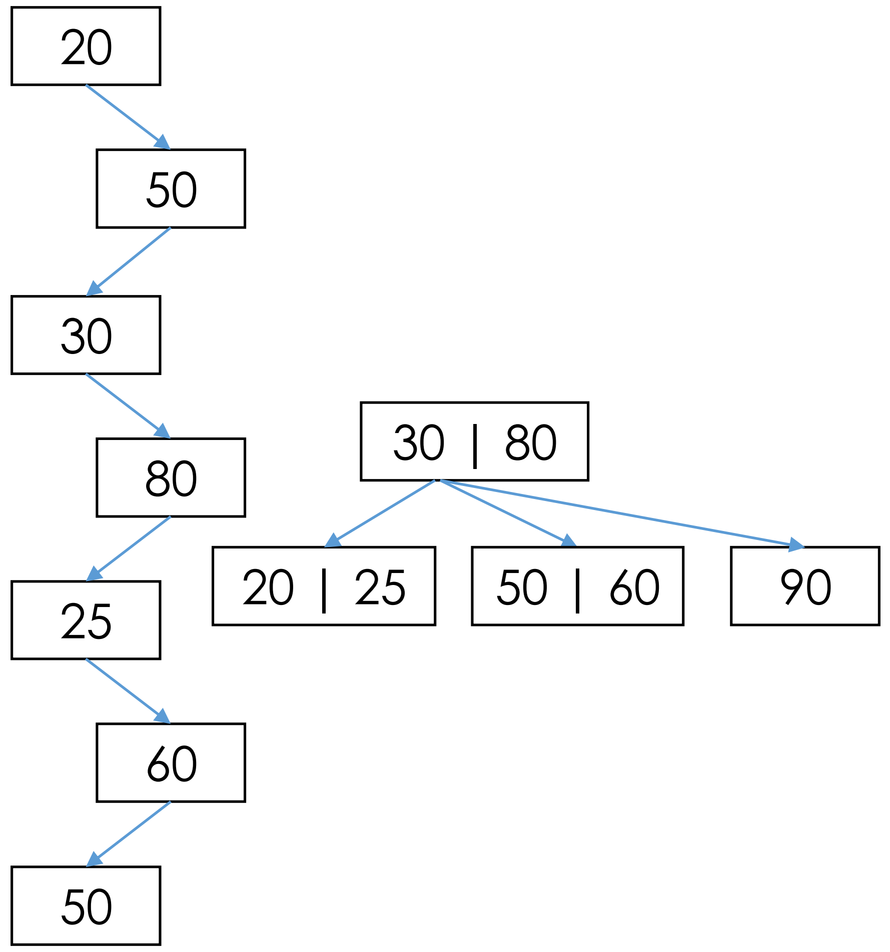

Removing From a 2-3 Tree

- Removing values from 2-3 trees
  - Always remove from a leaf
  - Values (and children) will be redistributed
  - Nodes can be merged
  - Only root node is deleted
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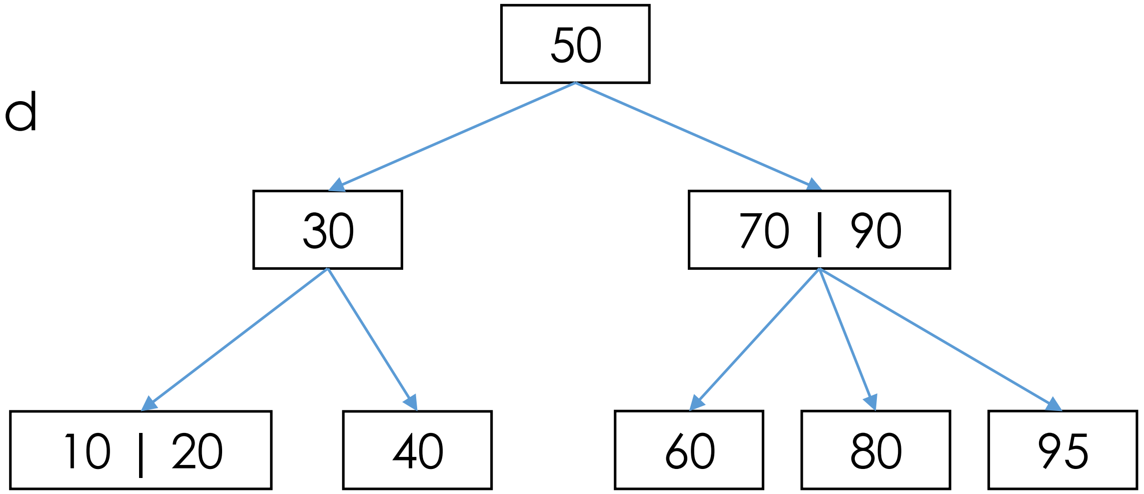

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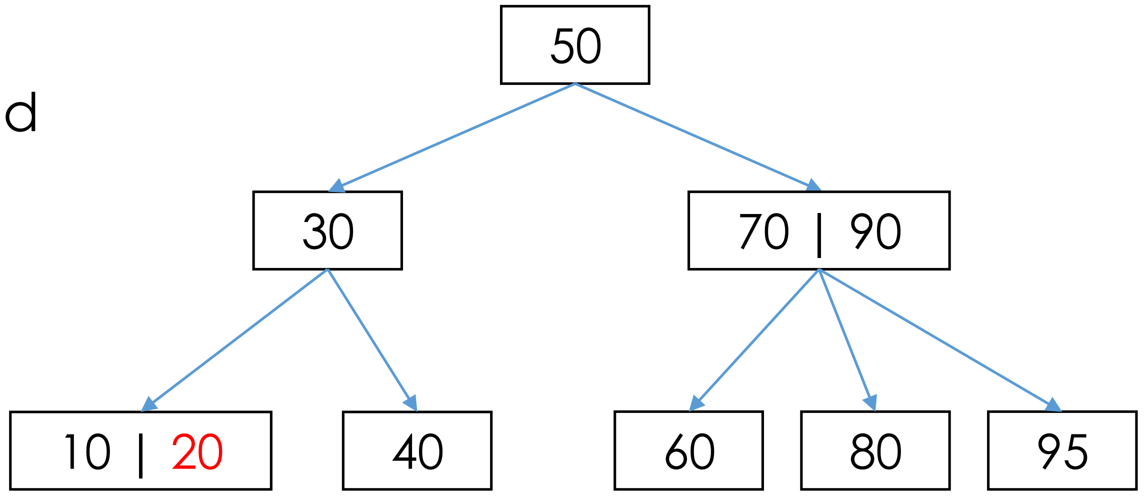

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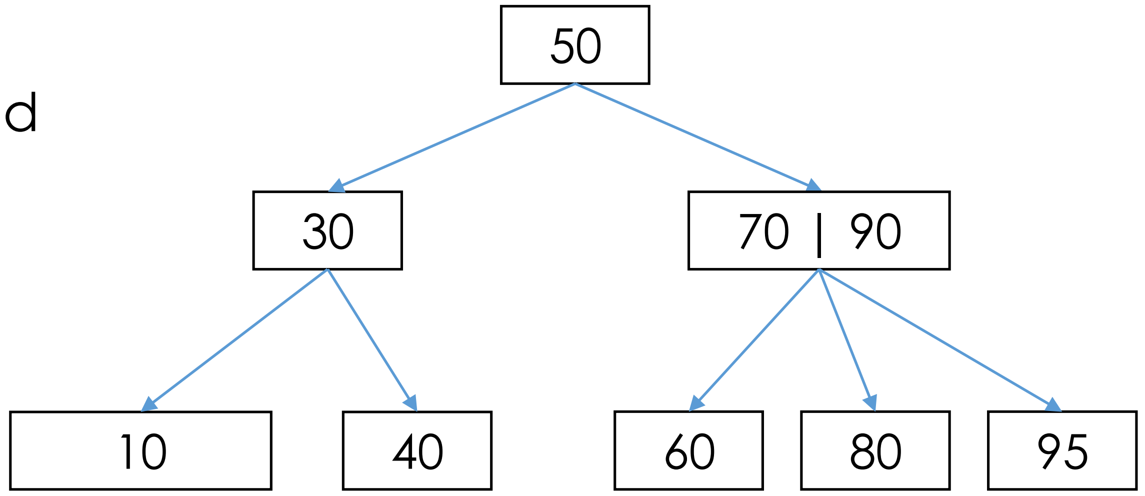

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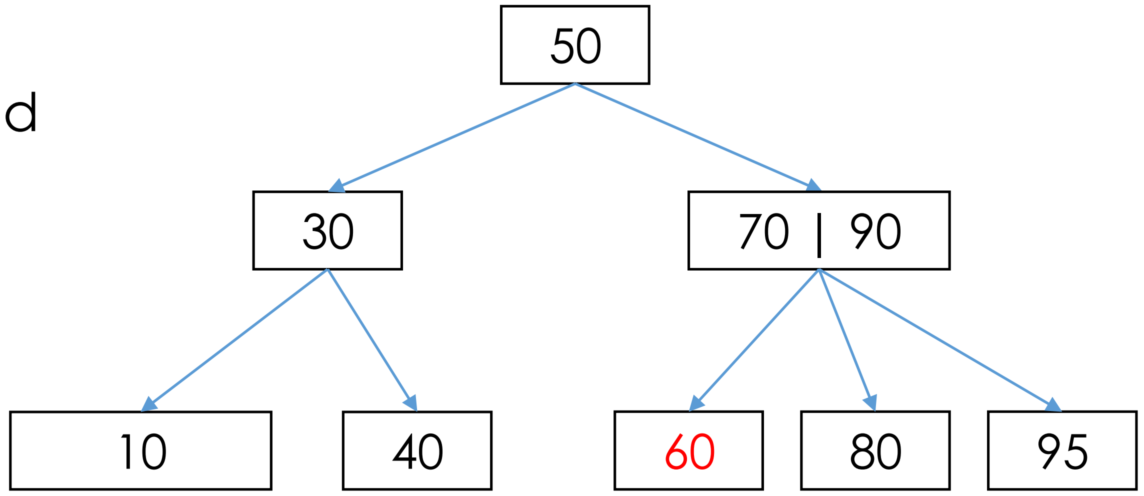

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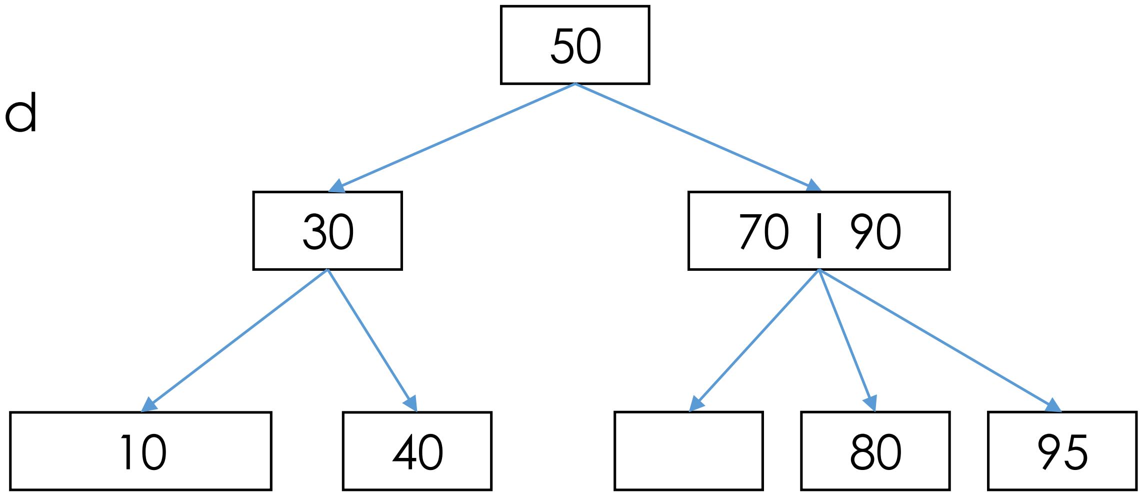

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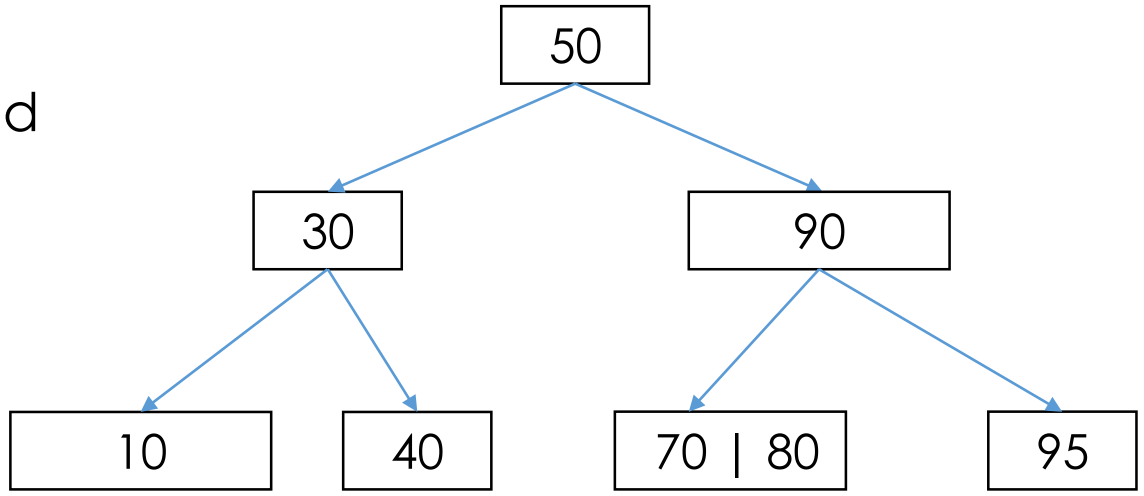

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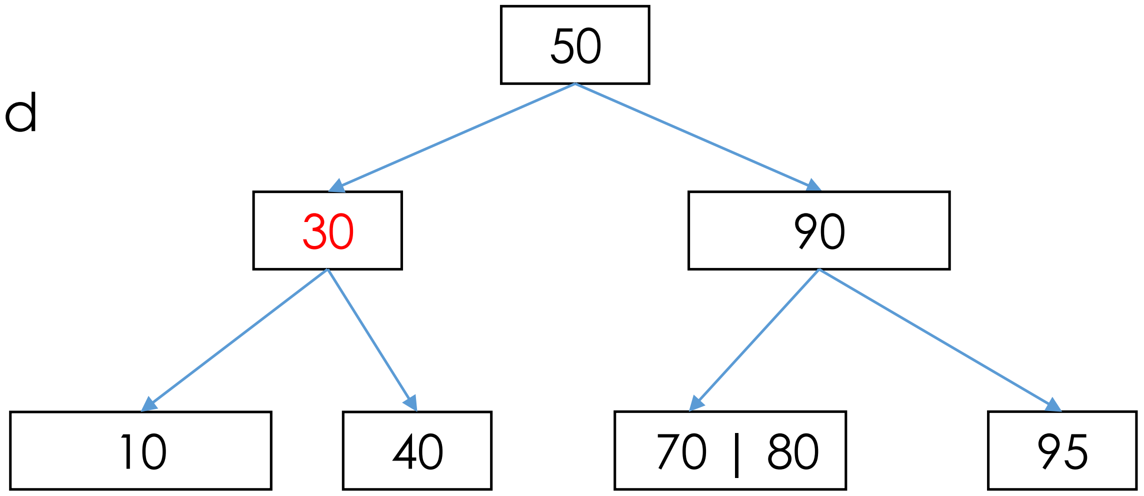

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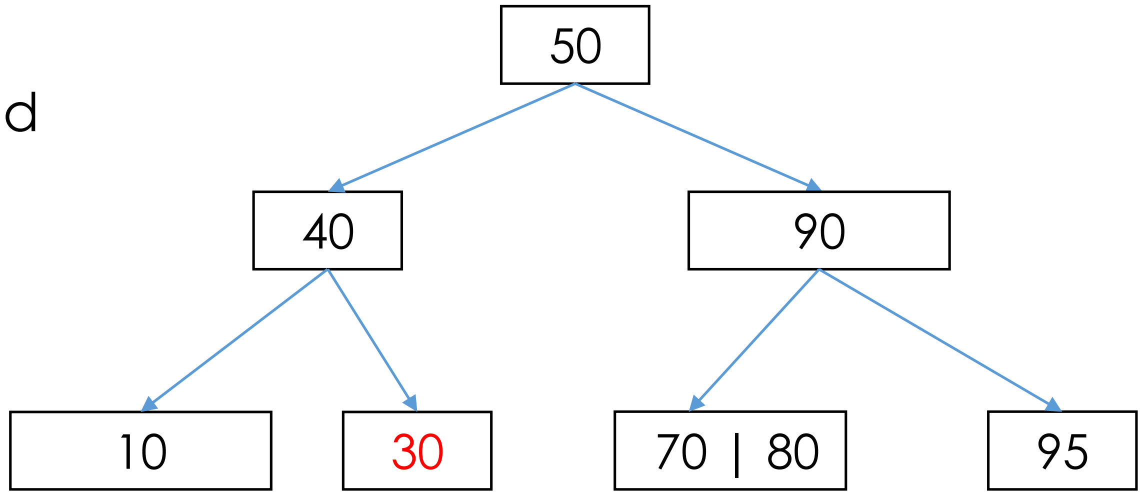
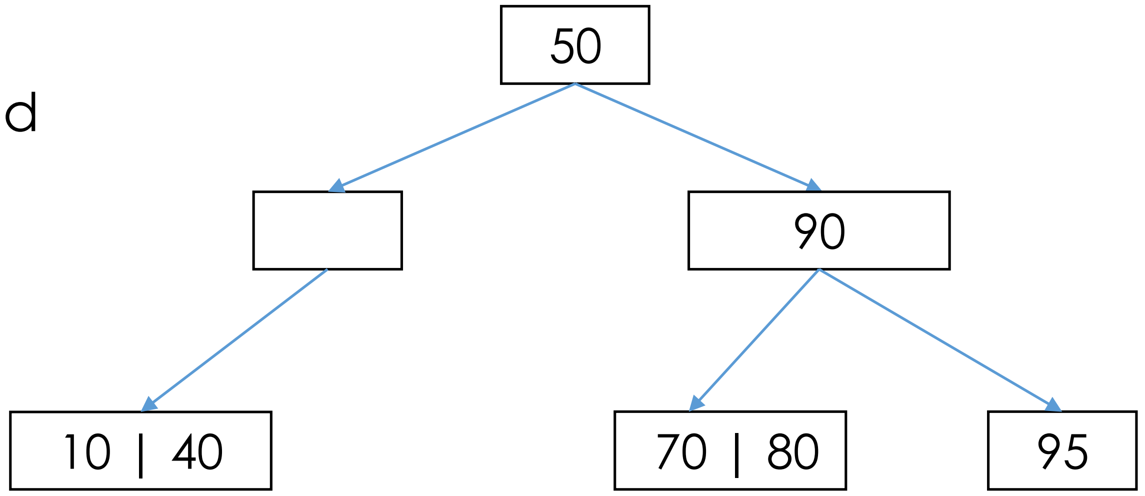


Removing From a 2-3 Tree

- Removing values from 2-3 trees
  - Always remove from a leaf
  - Values (and children) will be redistributed
  - Nodes can be merged
  - Only root node is deleted
    - And only if it is empty (contains no values)



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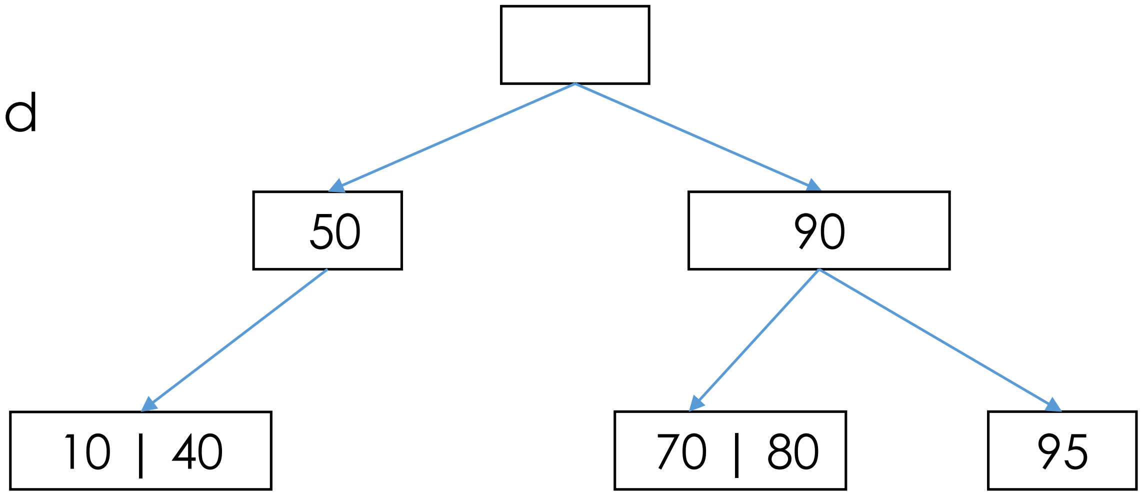

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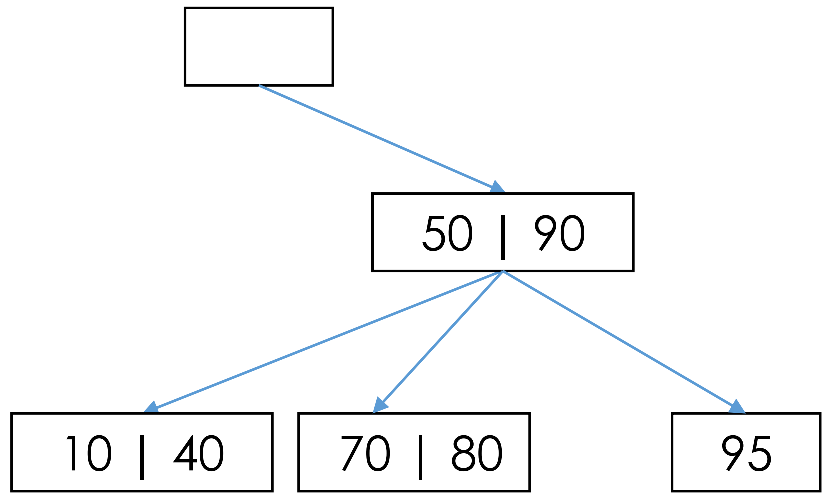

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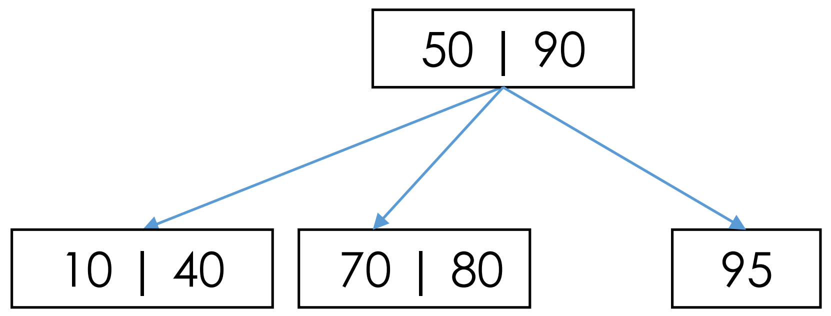

Thank you