CS302 - Data Structures using C++

Topic: Using the ADT Stack

Kostas Alexis



- Operator Precedence
 - Parenthesis ()
 - Exponentiation ^ (not a C++ expression)
 - Multiplication and Division * /
 - Addition and Subtraction + -

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20 - 2 * 8

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20 - 16

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- Binary Operators
 - Require two operands
 - 4 + 5
- Unary Operators
 - Single operand
 - - 6

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- Infix
- Common notation
 - 5+6
 - 5+6*7
 - (5+6)*7
- Prefix
- Functional languages
 - +56
 - + * 7 6 5
 - * + 5 6 7
- Postfix
- Reverse Polish Notation
 - 56+
 - 567*+
 - 756+*

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- To evaluate an infix expression
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression
- Use stacks to do so

Converting Infix Expressions to Postfix

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5*(4+2)^3

infix

postfix

Converting Infix Expressions to Postfix

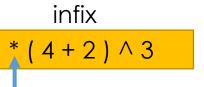
5*(4+2) ^ 3

infix

Operand – append to the postfix expression



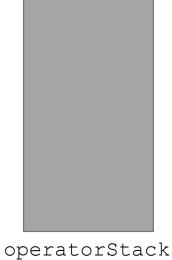
- Converting Infix Expressions to Postfix
 - Operand append to the postfix expression
 - Operators * / + etc
 - If operatorStack is not empty
 - Pop operators and append to the postfix expression
 - If their precedence >= that of the operator in the infix expression until operatorStack is empty or a (is reached
 - Push operator onto the operatorStack



postfix

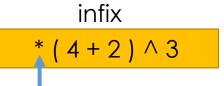
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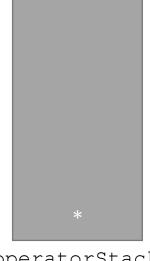


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postfix

5

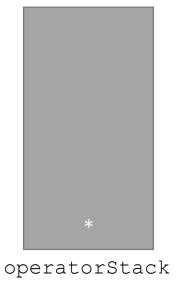


operatorStack



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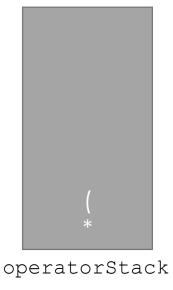
infix 4 + 2) ^ 3 postfix





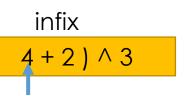
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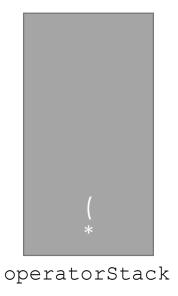




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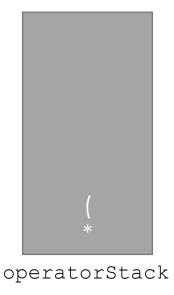




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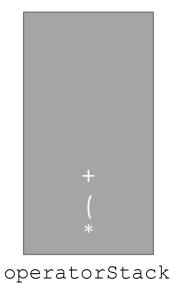
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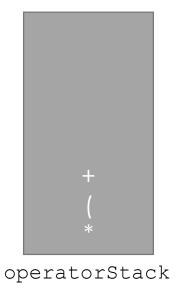






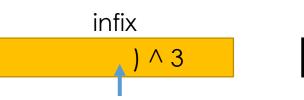
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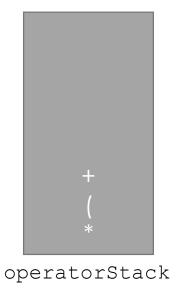




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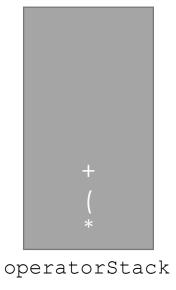
postfix 5 4 2





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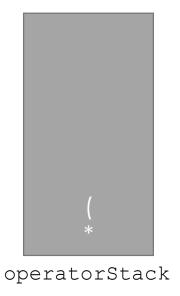






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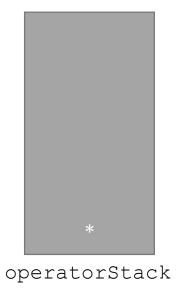






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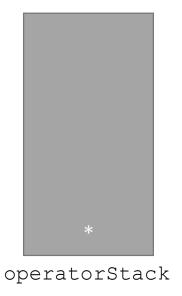






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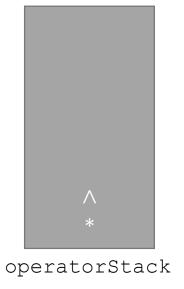






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infix

postfix

 $542 + 3 \land$



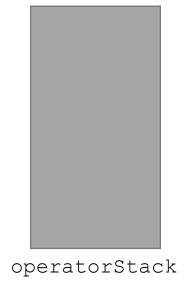


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postfix 5 4 2 + 3 ^ *

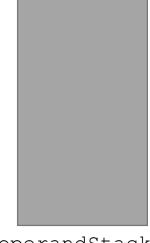


operandStack



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4 + 2



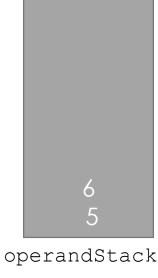




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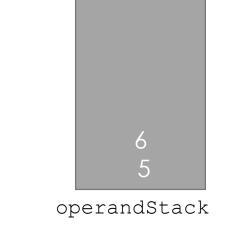
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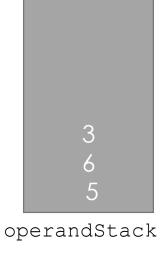
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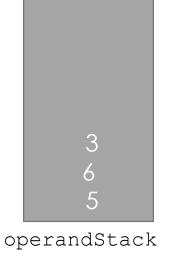
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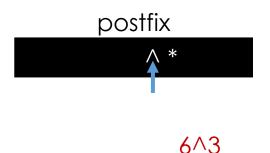


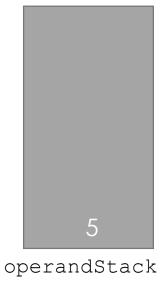
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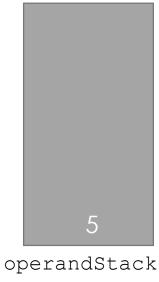




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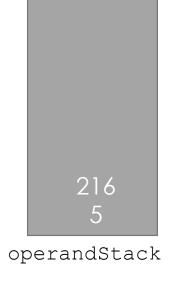


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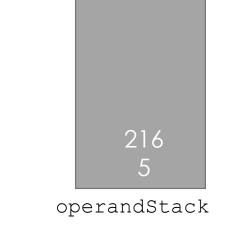
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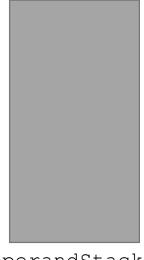




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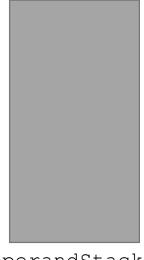


operandStack

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1080



operandStack



- To evaluate an infix expression
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression

```
Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression
infixExpression = infix expression to process
operatorStack = a new empty stack
postfixExpression = a new empty string
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- To evaluate an infix expression
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Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression
infixExpression = infix expression to process
operatorStack = a new empty stack
postfixExpression = a new empty string
while (infixQueue has characters left to parse)
           nextCharacter = next non-blank infixExpression character
           process (nextCharacter)
while (!operatorStack.isEmpty())
           topOperator = operatorStack.pop()
           append topOperator to postfixExpression
return postfixExpression
```

- To evaluate an infix expression
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression

```
// Process nextCharacter algorithm
switch (nextCharacter)
           case variable:
                      append nextCharacter to postfixExpression
           case \^/:
                      operatorStack.push (nextCharacter)
           case \(':
                      operatorStack.push (nextCharacter)
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                      append nextCharacter to postfixExpression
           case \^/:
                      operatorStack.push (nextCharacter)
           case \(':
                      operatorStack.push (nextCharacter)
           case ')':
                      topOperator = operatorStack.pop()
                      while(topOperator != '(')
                                 append topOperator to postfixExpression
                                 topOperator = operatorStack.pop()
           default:
                      break;
```

- To evaluate an infix expression
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression

```
append nextCharacter to postfixExpression
           case '^':
                      operatorStack.push (nextCharacter)
           case \(':
                      operatorStack.push (nextCharacter)
           case ')':
                      topOperator = operatorStack.pop()
                      while(topOperator != '(')
                                 append topOperator to postfixExpression
                                 topOperator = operatorStack.pop()
           case '+': case '-': case '*': case '/':
                      while(!operatorStack.isEmpty() and precedence of
nextCharacter <= precedence of operatorStack.peek())</pre>
                                  topOperator = operatorStack.pop()
                                 append topOperator to postfixExpression
                                 topOperator = operatorStack.pop()
                      peratorStack.push (nextCharacter)
           default:
                      break;
```

- To evaluate an infix expression
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression

```
Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression
infixExpression = infix expression to process
operatorStack = a new empty stack
postfixExpression = a new empty string
while (infixQueue has characters left to parse)
           nextCharacter = next non-blank infixExpression character
           process (nextCharacter)
while (!operatorStack.isEmpty())
           topOperator = operatorStack.pop()
           append topOperator to postfixExpression
return postfixExpression
```

- To evaluate an infix expression
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression

```
Algorithm evaluatePostfix(postfix)
// evaluate a postfix expression
valueStack = a new empty stack
while (postfixExpression has characters left to process)
             nextCharacter = next non blank character of postfixExpression
             switch (nextCharacter)
                          case variable:
                                       valueStack.push(value of var nextCharacter)
                                       break
                          case '+': case '-': case '*'; case '/': case '*':
                                       operandTwo = valueStack.peek()
                                       valueStack.pop()
                                       operandOne = valueStack.peek()
                                       valueStack.pop()
                                       result = apply operation in nextCharacter to its
operands operandOne and operandTwo
                                       break
                          default: break
             } // end switch
} // end while
```

- To evaluate an infix expression
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression

```
Algorithm evaluatePostfix(postfix)
// evaluate a postfix expression
valueStack = a new empty stack
while (postfixExpression has characters left to process)
             nextCharacter = next non blank character of postfixExpression
             switch (nextCharacter)
                          case variable:
                                       valueStack.push(value of var nextCharacter)
                                       break
                          case '+': case '-': case '*'; case '/': case '*':
                                       operandTwo = valueStack.peek()
                                       valueStack.pop()
                                       operandOne = valueStack.peek()
                                       valueStack.pop()
                                       result = apply operation in nextCharacter to its
operands operandOne and operandTwo
                                       break
                          default: break
             } // end switch
} // end while
return valueStack.peek()
```

Thank you

