# CS302 - Data Structures using C++ 

Topic: Using the ADT Stack

Kostas Alexis

## Algebraic Expression

- Operator Precedence
- Parenthesis ()
- Exponentiation $\wedge$ (not a C++ expression)
- Multiplication and Division */
- Addition and Subtraction + -


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- 4 + 5
- Unary Operators
- Single operand
-     - 6


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- Infix
- Common notation
- $5+6$
- $5+6$ * 7
- $(5+6) * 7$
- Prefix
- Functional languages
-     + 56
-     +         * 765
-     * +567
- Postfix
- Reverse Polish Notation
- $56+$
- $567^{*}+$
- 756 +*


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## Evaluating Infix Expressions

- To evaluate an infix expression
- Convert the infix expression to postfix form
- Evaluate the postfix expression
- Use stacks to do so


## Evaluating Infix Expressions

- Converting Infix Expressions to Postfix


## Evaluating Infix Expressions

$\begin{array}{cc} & \text { infix } \\ \text { - Converting Infix Expressions to Postfix } & 5 *(4+2) \wedge 3\end{array}$

## Evaluating Infix Expressions

- Converting Infix Expressions to Postfix
- infix
Operand - append to the postfix
expression


## Evaluating Infix Expressions

- Converting Infix Expressions to Postfix
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- Operand - append to the postfix expression
- Operators * / + - etc
- If operatorStack is not empty
- Pop operators and append to the postfix expression
- If their precedence >= that of the operator in the infix expression until operatorStack is empty or a (is reached
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- Scan characters in the postfix expression
- When an operand is entered
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## Evaluating Infix Expressions

- To evaluate an infix expression
- Convert the infix expression to postfix form
- Evaluate the postfix expression

Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression
infixExpression $=$ infix expression to process
operatorStack $=$ a new empty stack
postfixExpression $=$ a new empty string

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Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression
infixExpression = infix expression to process
operatorStack = a new empty stack
postfixExpression = a new empty string
while (infixQueue has characters left to parse)
{
    nextCharacter = next non-blank infixExpression character
    process(nextCharacter)
}
while (!operatorStack.isEmpty())
{
    topOperator = operatorStack.pop()
    append topOperator to postfixExpression
}
return postfixExpression
```


## Evaluating Infix Expressions

- To evaluate an infix expression
- Convert the infix expression to postfix form
- Evaluate the postfix expression

```
        Process nextCharacter algorithm
    switch (nextCharacter)
                                    case variable:
                            append nextCharacter to postfixExpression
    case
                            operatorStack.push(nextCharacter)
case '(':
operatorStack.push(nextCharacter)
```


## Evaluating Infix Expressions

- To evaluate an infix expression
- Convert the infix expression to postfix form
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## Evaluating Infix Exp̈ressions <br> case variable:

- To evaluate an infix expression
- Convert the infix expression to postfix form
- Evaluate the postfix expression
append nextCharacter to postfixExpression
 append topOperator to postfixExpression topOperator $=$ operatorStack.pop()
case '+': case '-': case '*': case '/':
while(!operatorStack.isEmpty() and precedence of nextCharacter $<=$ precedence of operatorStack.peek())

opoperator $=$ operatorStack.pop()<br>append topOperator to postfixExpression

topOperator $=$ operatorStack.pop()
peratorStack.push(nextCharacter)
default:
break;

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}
return postfixExpression
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## Evaluating Infix Expressions

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- Convert the infix expression to postfix form
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Algorithm evaluatePostfix(postfix
// evaluate a postfix expression
valueStack = a new empty stack
while(postfixExpression has characters left to process)
nextCharacter = next non blank character of postfixExpression
switch (nextCharacter)
case variable:
                                    valueStack.push(value of var nextCharacter)
                                    break
                                    case '+': case '-': case '*'; case '/': case '*':
                            operandTwo = valueStack.peek()
                            valueStack.pop()
                            operandOne = valueStack.peek()
                            valueStack.pop()
                            result = apply operation in nextCharacter to its
                            operands operandOne and operandTwo
                            break
default: break
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                                    valueStack.pop()
                                    result = apply operation in nextCharacter to its
                                    operands operandOne and operandTwo
                                    break
default: break
} // end while
return valueStack.peek()
```

Thank you

