CS302 - Data Structures

using C++

Topic: Examples

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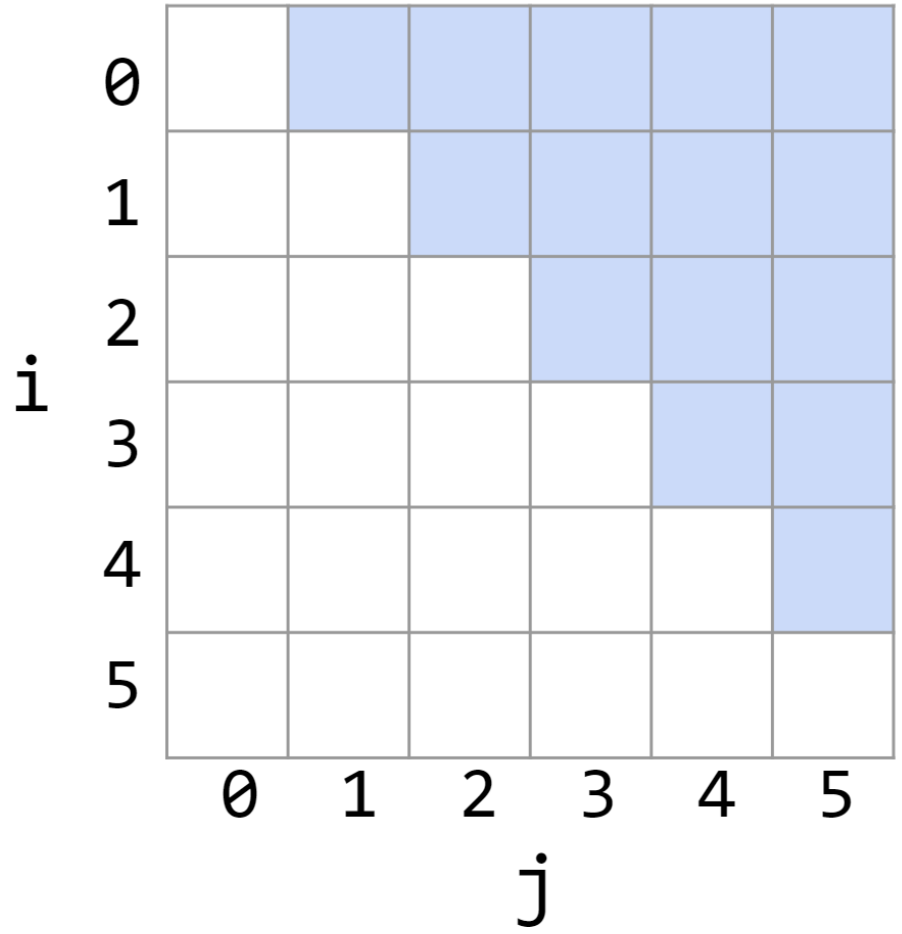
AUTONOMOUS ROBOTS LAB

Summary of Asymptotic Notations

	Informal meaning:	Family	Family Members
Big Theta $\Theta(f(N))$	Order of growth is f(N).	$\Theta(N^2)$	$N^2/2$ $2N^2$ $N^2 + 38N + N$
Big O $O(f(N))$	Order of growth is less than or equal to f(N).	$O(N^2)$	$N^2/2$ $2N^2$ $\lg(N)$
Big Omega $\Omega(f(N))$	Order of growth is greater than or equal to f(N).	$\Omega(N^2)$	$N^2/2$ $2N^2$ e^N

Example: Nested For Loops

- Find a simple $f(N)$ such that the runtime $R(N) \in \Theta(f(N))$ in the worst case

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int N = length;
for (int i = 0; i < N; i++ 1)
    for (int j = 1; j < N; j++)
          if (a[i]==a[j])
              return true;
return false;
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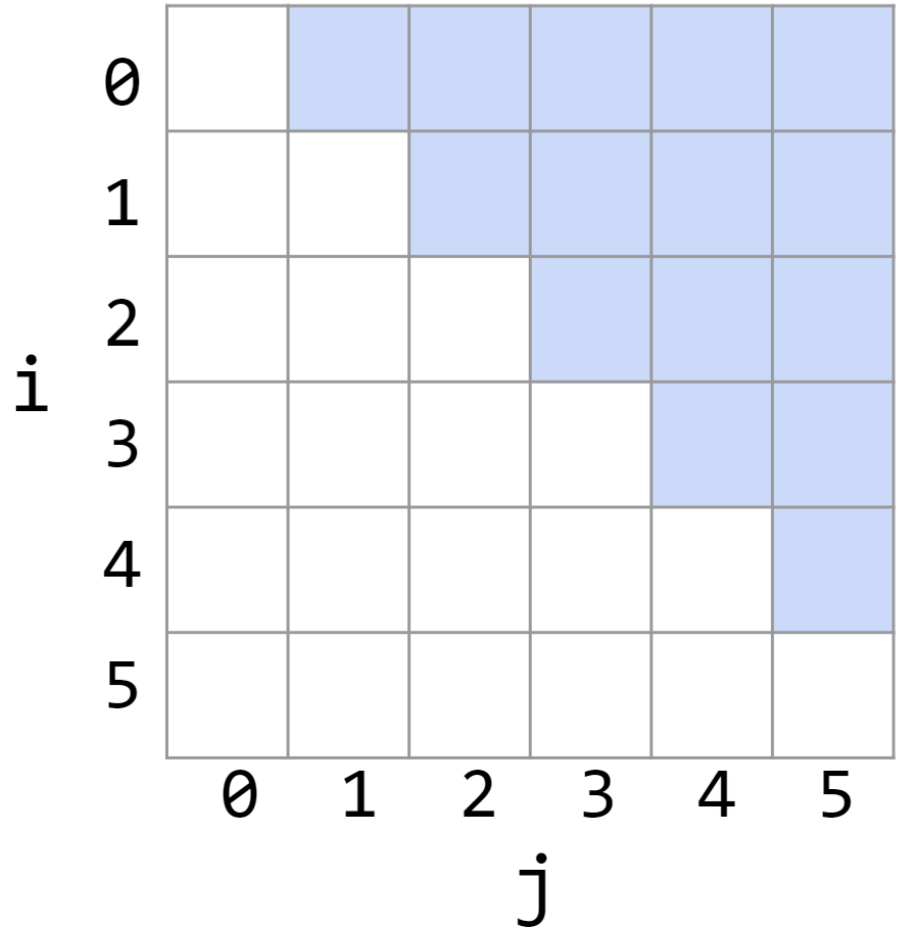


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- Worst case numbers of j++ calls
- 1+2+3+…+(N-3)+(N-2)+(N-1)=**N(N-1)/2**

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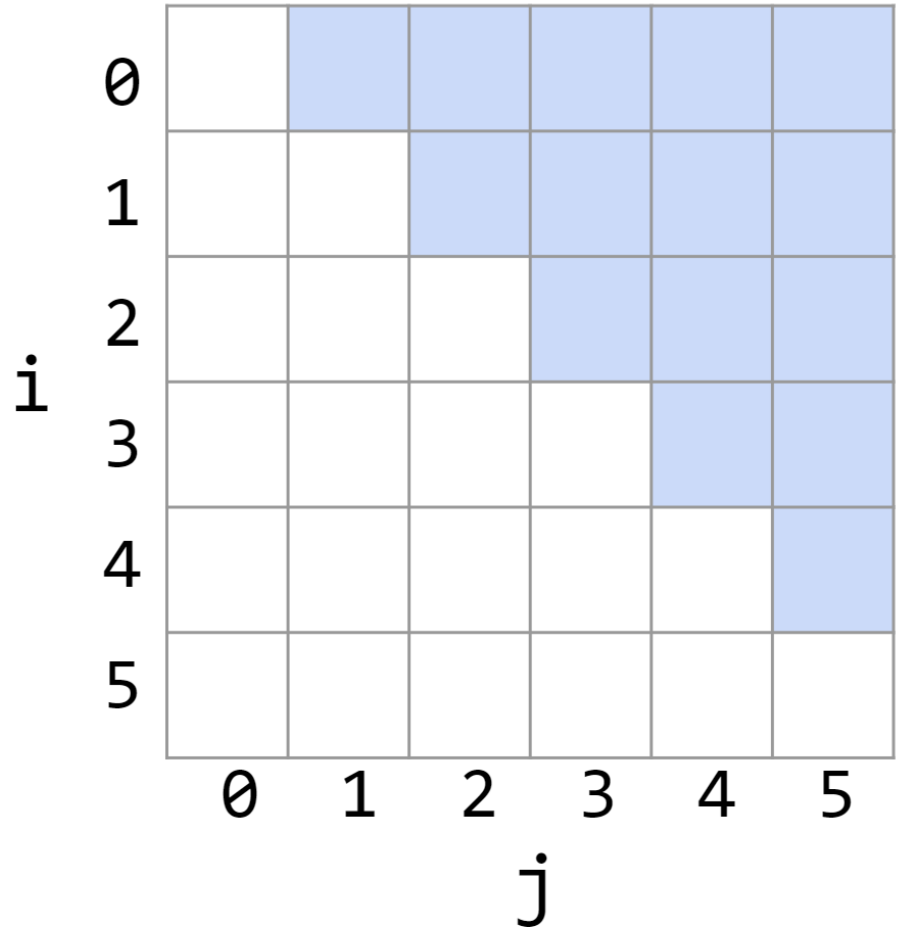


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- **Overall worst case runtime: $\Theta(N^2)$**

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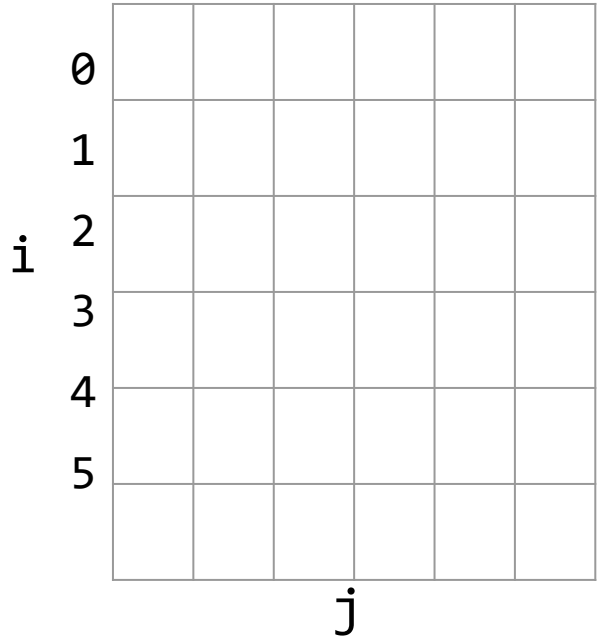


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public static void printIndices(int n){
    for (int i = 0; i < n; i=i*2){
          for (int j = 0; i < N; j++)
                cout<<"hello" << endl;
                int A=1+1;
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- Cost model, cout calls

1	**2**	3	**4**	5	6	7	**8**	9	10	11	12	13	14	15	**16**	17	18	19	20

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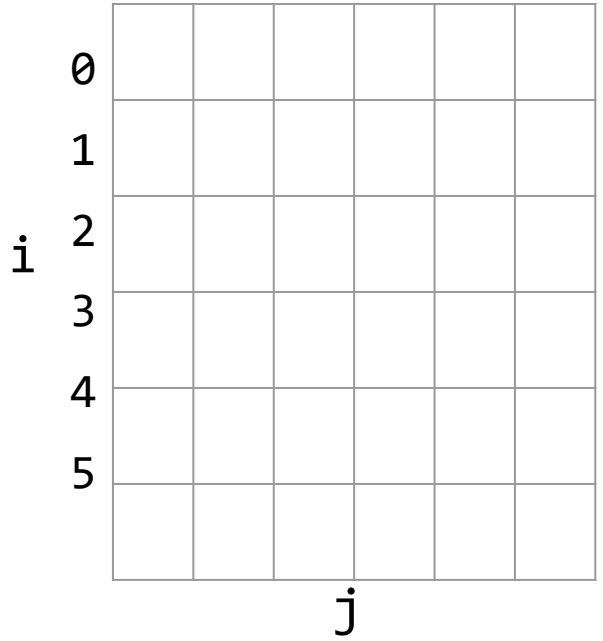


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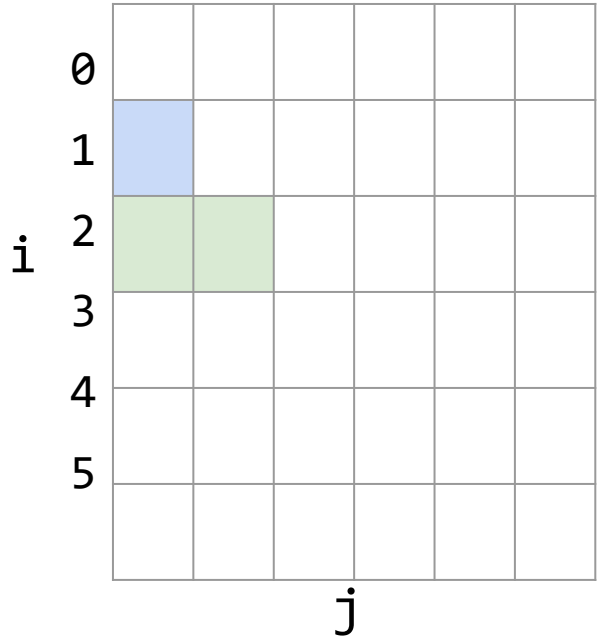


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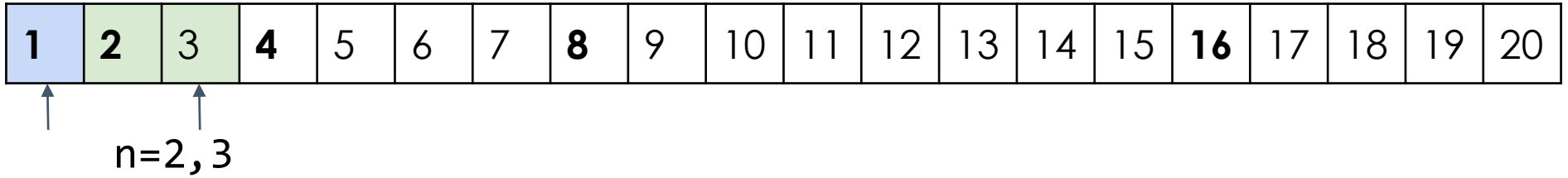

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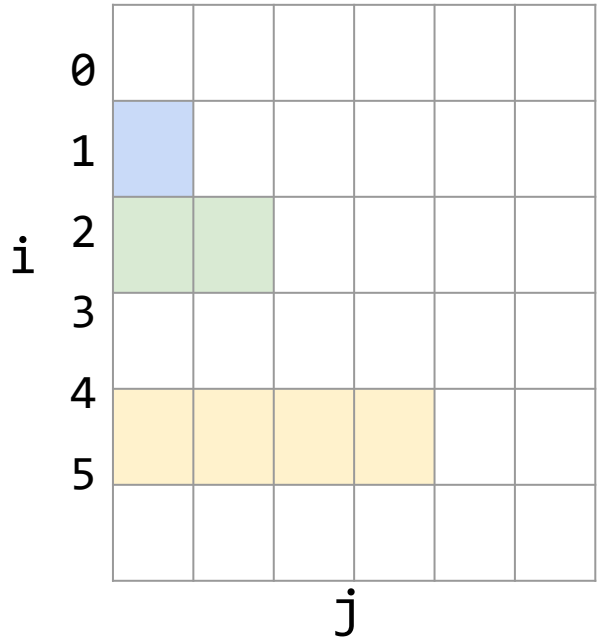


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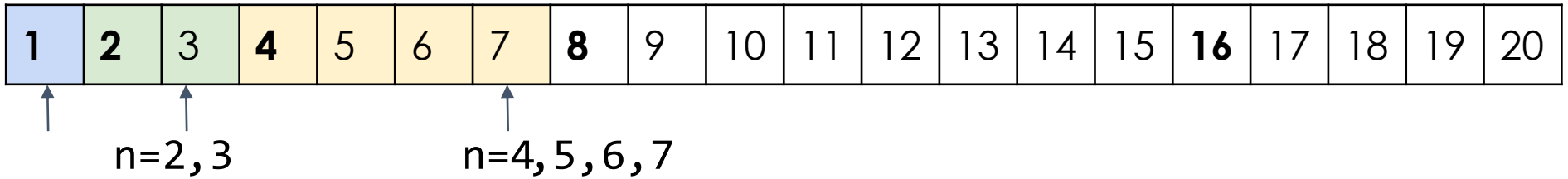

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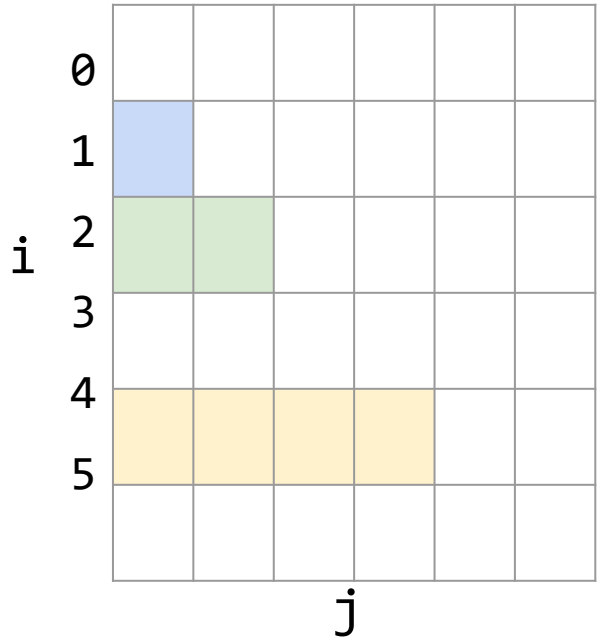


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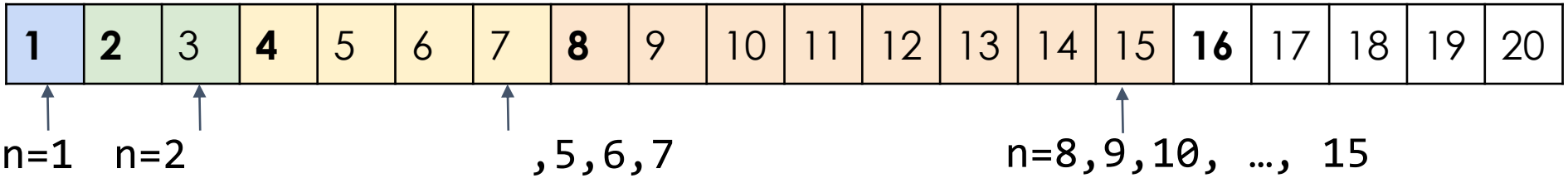

Example: Nested For Loops II

- Find a simple $f(N)$ such that the runtime $R(N) \in \Theta(f(N))$
- Worst case here is irrelevant, all cases the same
- Cost model calls R(N) = Θ(1+2+4+8+…+N)

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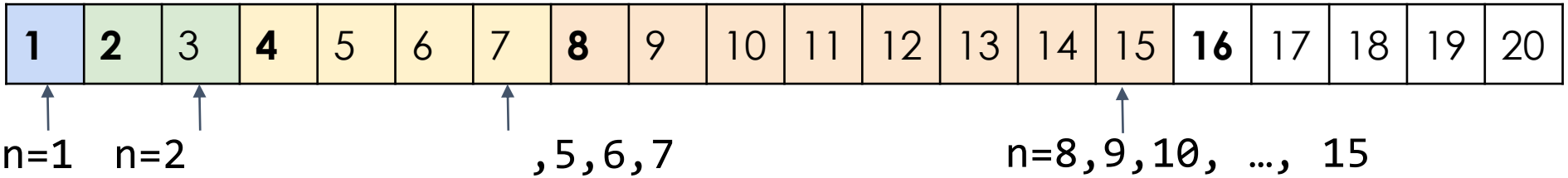
Cases

A. 1
B. logN
C. N
D. NlogN
E. N^2
F. Other



Example: Nested For Loops II

- Find a simple $f(N)$ such that the runtime $R(N) \in \Theta(f(N))$

N	R(N)	1 * N < R(N)	2 * N > R(N)
1	1	1	2
4	1 + 2 + 4 = 7	4	8
7	1 + 2 + 4 = 7	7	14
8	1 + 2 + 4 + 8 = 15	8	16
27	1 + 2 + 4 + 8 + 16 = 31	27	54
185	… + 64 + 128 = 255	185	370
715	… + 256 + 512 = 1023	715	1430

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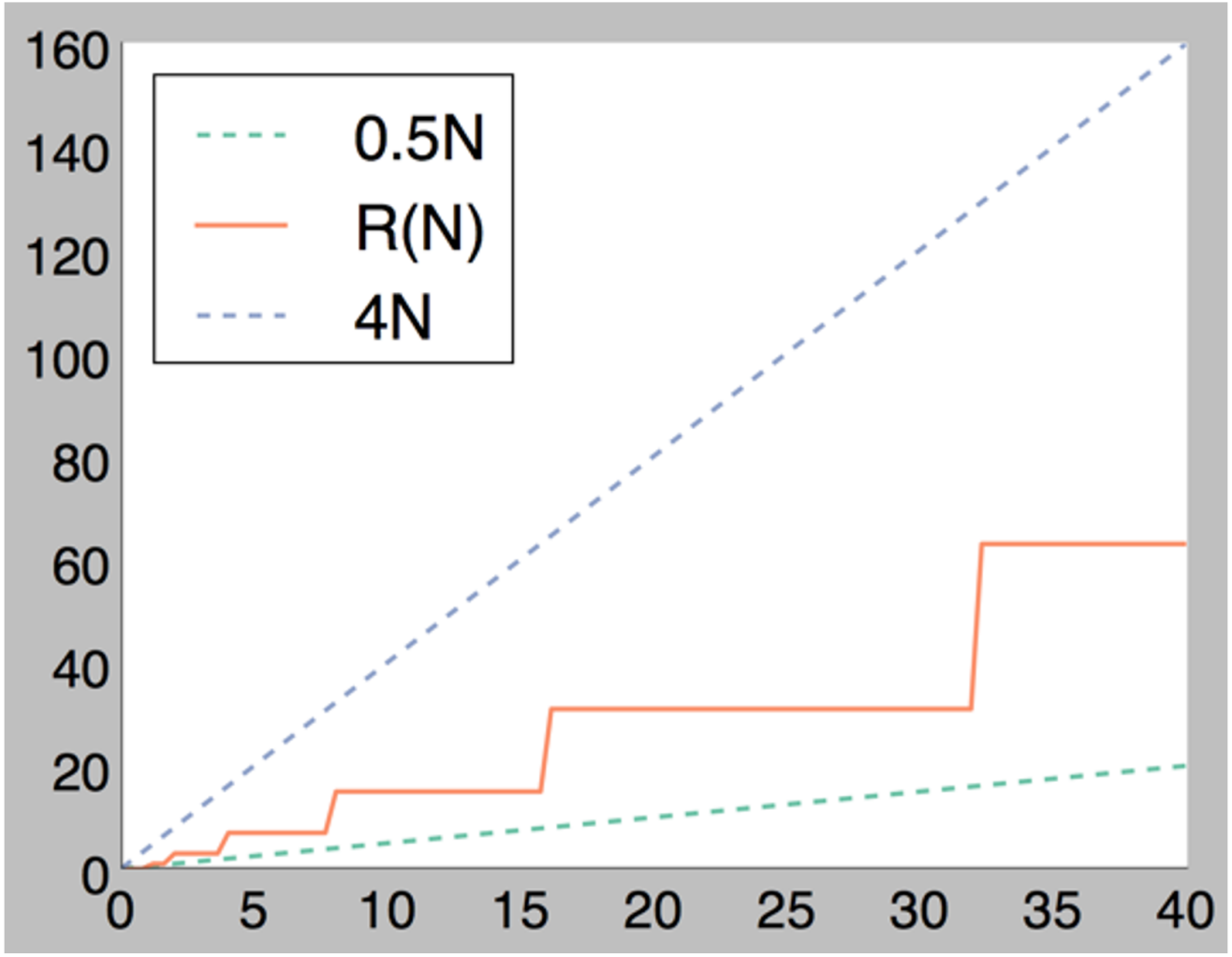

$R(N) = \Theta(1 + 2 + 4 + 8 + \ldots + N)$

$= \Theta(N)$

A. 1
B. log N
C. N
D. N log N
E. N^2
F. Something else

Thank you