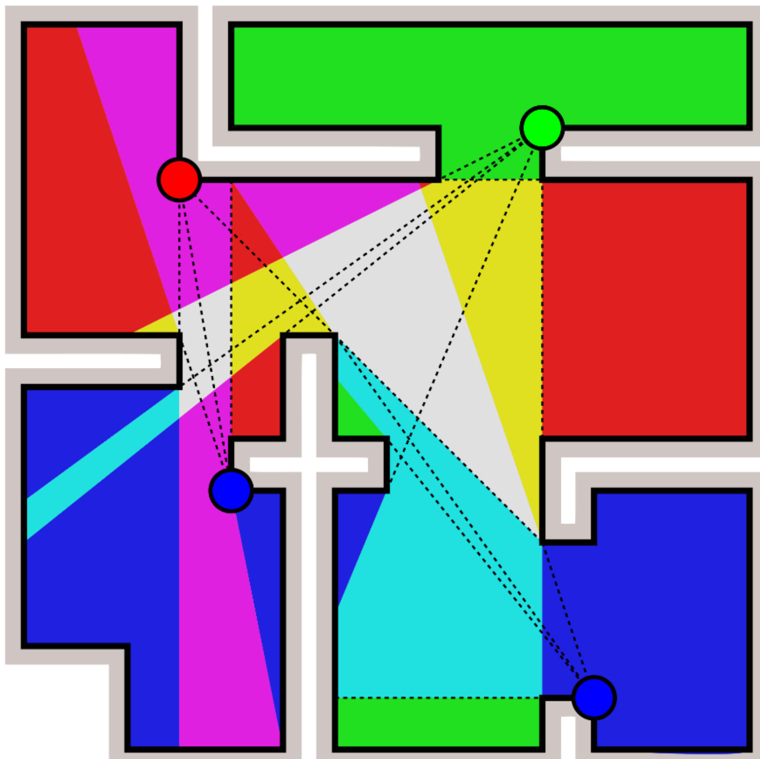
CS302: Data Structures – Bonus Assignment 2018 #2

Theme: The Art Gallery Problem



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Section 1: Introduction to the Art Gallery Problem

The Art Gallery Problem (AGP) is a well-studied problem in computational geometry [1-5]. AGP originates from the real-world task of guarding an art gallery with the minimum number of guards such that complete coverage is continuously ensured. In the rather simplified geometric version of the problem, the layout of the "art gallery" is represented by a simple polygon and each guard is a point in this polygon. A set S of points is said to guard a polygon if, for every point p in that polygon, there is some q such that the line segment between p and $q \in S$ does not leave the polygon. The AGP may not only be defined in 2D but also in 3D. Its use-cases (directly or indirectly) involve a variety of important tasks including – but not limited to – security surveillance and associated sensor place, searching terrains, robotic inspection path planning and more [6-8]. For the remainder of this assignment document, we will constrain our study into the 2D case.

The problem defined through an example: Given the floor plan of a weirdly shaped art gallery having N straight sides, how many guards will we need to post, in the worst case, so that every bit of wall is visible to a guard?

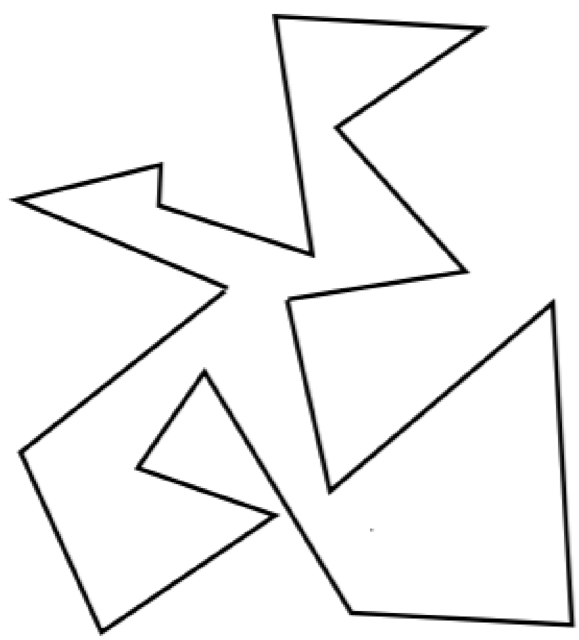

Figure 1. An example "art gallery" – every wall of it must be guarded.

The result is that we can ensure full "guarding" with N/3 "guards". To understand this, and also clarify what are the assumptions for each "guard", let us see some examples:

- A convex gallery needs only one guard (Figure 2.a)
- A star-shaped gallery needs only one guard (Figure 2.b)
- A 4-sided gallery needs only one guard (Figure 2.c)
- A 5-sided gallery needs only one guard (Figure 2.d)

However:

- A 6-sided gallery might need two guards (Figure 2.e)
- An N-sided gallery might need N/3 guards (Figure 2.f)

In fact, a comb is the wost-case (e.g., Figure 2.f).

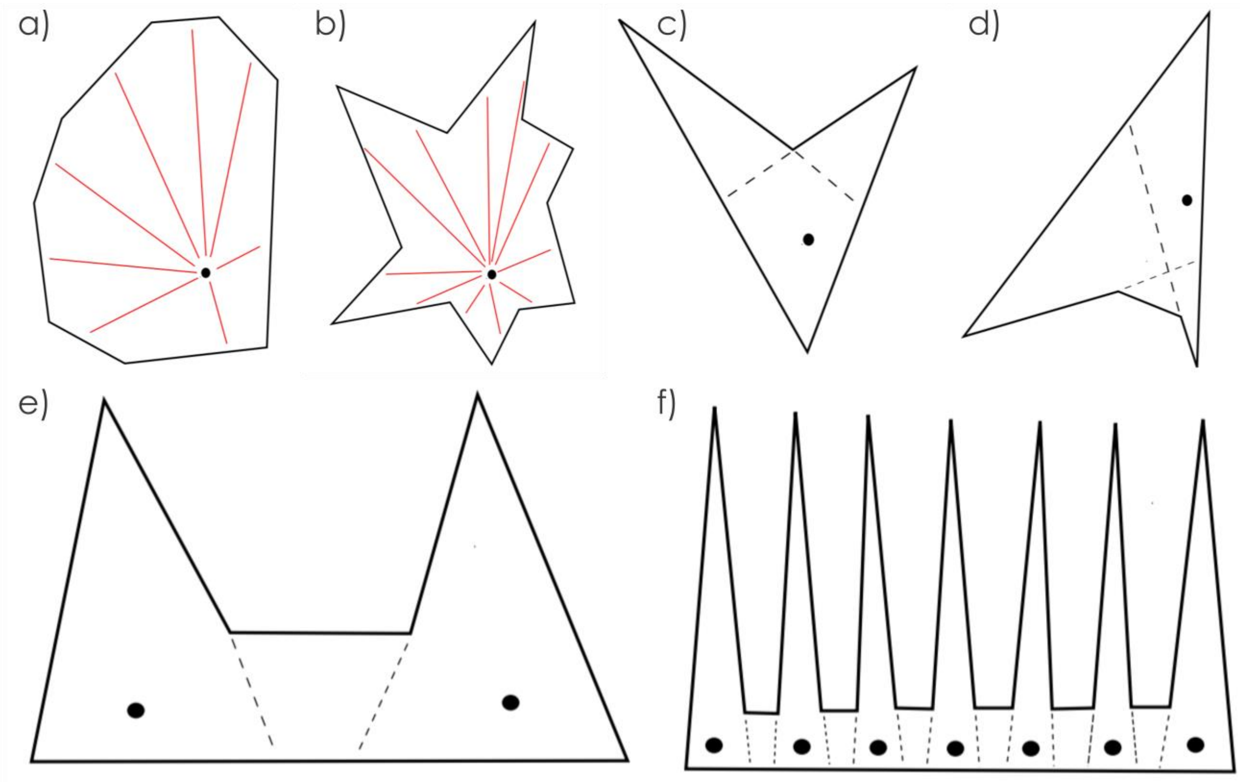

Figure 2. Cases of "galleries" and the required amount of guards.

Assisted by the above discussion, we can now pose the problem and its solution:

Problem (V. Klee, 1973): How many guards does an N-sided gallery need? Is the comb the worst case?

Theorem (V. Chvatal, 1973, shortly thereafter): Yes, the combs achieve the worst-case: every N-sided gallery needs at most N/3 guards.

A Methodology to solve AGP: In this subsection we will discuss a simple methodology to solve the AGP. Many more approaches have been proposed. Assuming the gallery in Figure 3.a, the sequence of steps to follow are as follows:

1. Triangulate the gallery without new vertices (Figure 3.b)
2. Properly 3-color its vertices (Figure 3.c)
3. Note that the least popular color gets used at most N/3 times
4. Post guards near the least popular color vertices (Figure 3.d)

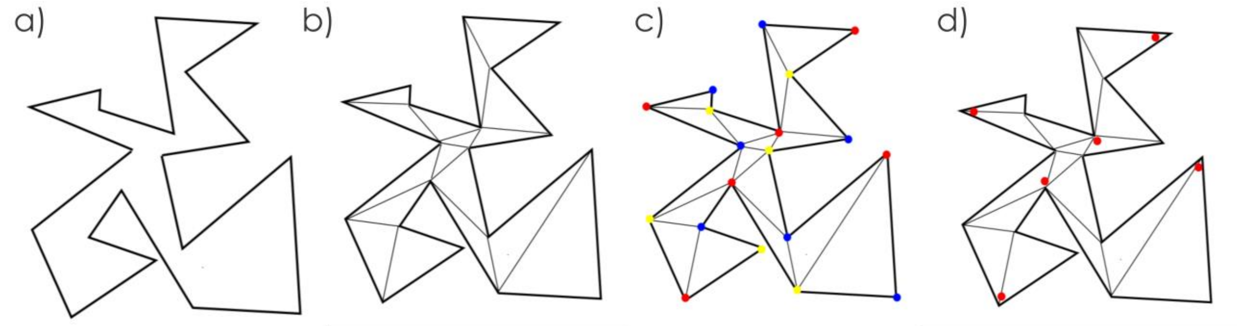

Figure 3. Basic steps to solve the AGP.

Of particular importance is the best to triangulate without new vertices which is achieved by using induction steps as shown in Figure 4.a, while similarly we can color by induction (Figure 4.b) and if needed glue back the colorings together (Figure 4.c). With respect to how to start drawing extra "lines" for triangulation, the utilized concept is the following:

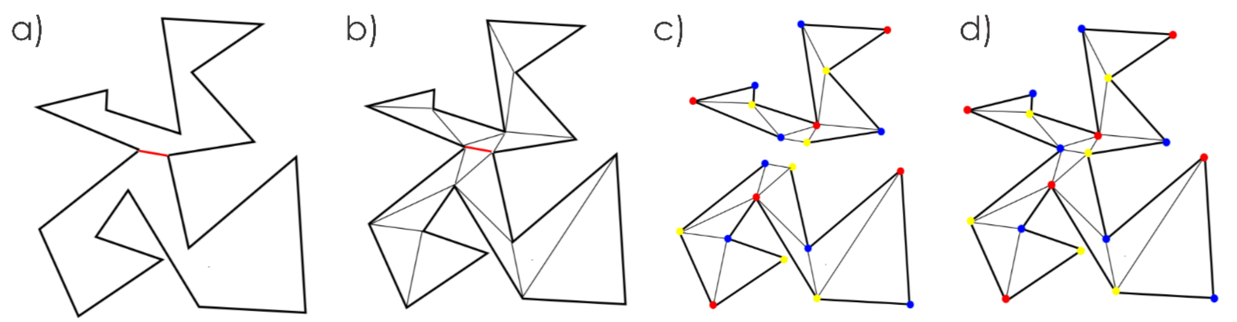

Figure 4. How to draw the "red" line for triangulation with existing vertices.

Flashlight argument: Starting a vertex X, shine a flashlight along the wall to an adjacent vertex Y, and swing it in an arc until you first hit another vertex Z. Then either XZ or YZ works as the "red" dividing line as shown in Figure 5.

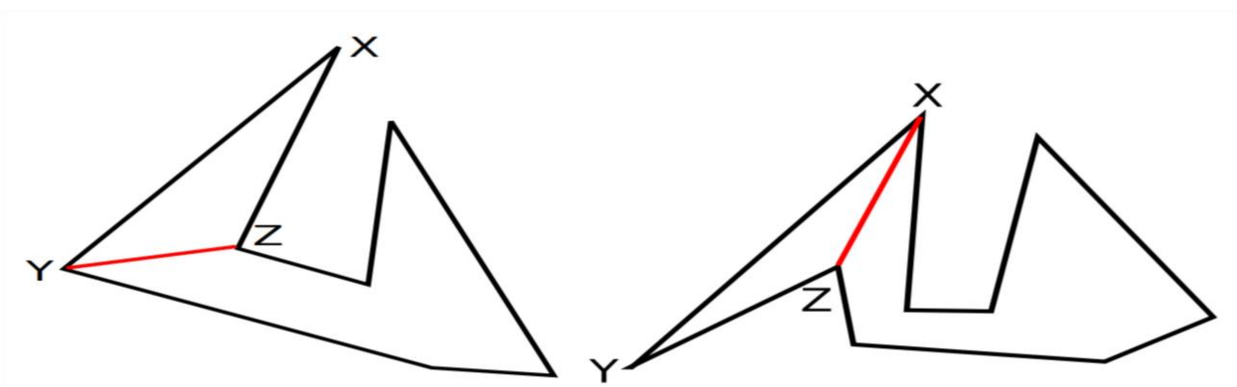

Figure 5. The "Flashlight" argument.

This is one solution to the problem and provides a true solution only subject to careful implementation as best visualized in Figure 6. In fact, the quality of the result depends on the triangulation.

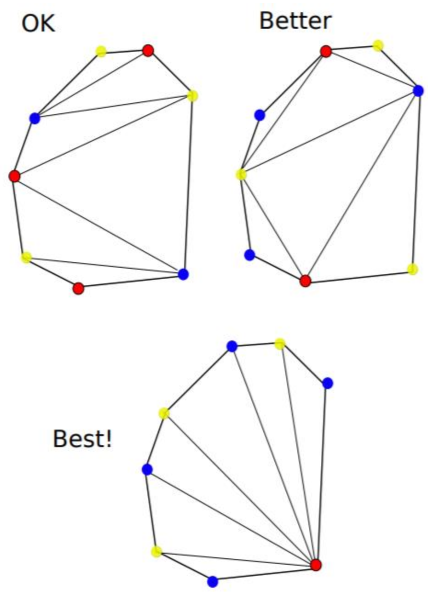

Figure 6. The solution of the AGP with the presented method depends on the triangulation.

Section 2: Programming Assignment

Your assignment is to implement a solution for 2D Art Gallery Problems assuming a polygon representation and guard operation such as the one presented above. The implementation must be in C++ and your report should outline the algorithm and explain the graph concepts you had to use to realize it.

Section 8: What now?

If you liked this assignment, or if you found it too easy and want to explore more, please do one of the following:

- Contact us at kalexis@unr.edu or by talking to any other member of the Autonomous Robots Lab (www.autonomousrobotslab.com)
- Find out projects we propose for students here https://www.autonomousrobotslab.com/student-projects2.html or design one on your own and let us know for your ideas. If you like these problems emphasize on path planning problems for robotics.
- Build a community of undergrads in robotics – join "BadgerWorks": https://www.autonomousrobotslab.com/badgerworks.html

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