



CS491/691: Introduction to Aerial Robotics

Topic: Time Derivatives in 2 Frames

Dr. Kostas Alexis (CSE)

Time Derivatives in 2 Frames

- ▶ Time Derivatives in a Rotating Frame:

- ▶ Introduce the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ representing standard unit basis vectors in the rotating frame. As they rotate they will remain normalized. If we let them rotate at a speed Ω about an axis Ω then each unit vector \mathbf{u} of the rotating coordinate system abides by the rule:

$$\frac{d}{dt}\mathbf{u} = \Omega \times \mathbf{u}$$

- ▶ Then if we have a unit vector:

$$\mathbf{f}(t) = f_x(t)\mathbf{i} + f_y(t)\mathbf{j} + f_z(t)\mathbf{k}$$

- ▶ To examine its first derivative – we have to use the product rule of differentiation:

$$\frac{d}{dt}\mathbf{f} = \frac{df_x}{dt}\mathbf{i} + \frac{d\mathbf{i}}{dt}f_x + \frac{df_y}{dt}\mathbf{j} + \frac{d\mathbf{j}}{dt}f_y + \frac{df_z}{dt}\mathbf{k} + \frac{d\mathbf{k}}{dt}f_z \Rightarrow$$

$$\frac{d}{dt}\mathbf{f} = \left[\left(\frac{d}{dt} \right)_r + \Omega \times \right] \mathbf{f}$$

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Time Derivatives in 2 Frames

- ▶ As a result: Relation Between Velocities in the Inertial & Rotating Frame

- ▶ Let \mathbf{r} be the position of an object's position:

$$\mathbf{v} = \frac{d}{dt}\mathbf{p}$$

- ▶ Then the relation of the velocity as expressed in the inertial frame and as expressed in the rotating frame becomes:

$$\mathbf{v}_i = \mathbf{v}_r + \boldsymbol{\Omega} \times \mathbf{p}$$

- ▶ Similarly: Relation Between Accelerations in the Inertial & Rotating Frame

- ▶ Let \mathbf{a} be the acceleration of an object's position. Then:

$$\mathbf{a}_i = \left(\frac{d\mathbf{p}}{dt}\right)_i = \left(\frac{d\mathbf{v}}{dt}\right)_i = \left[\left(\frac{d}{dt}_r + \boldsymbol{\Omega} \times\right)\right] \left[\left(\frac{d\mathbf{p}}{dt}\right)_r + \boldsymbol{\Omega} \times \mathbf{p}\right]$$

- ▶ Carrying out the differentiations:

$$\mathbf{a}_r = \mathbf{a}_i - 2\boldsymbol{\Omega} \times \mathbf{v}_r - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{p}) - \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{p}$$

- ▶ Subscripts i, r represent the inertial frame and the rotating frame respectively.

A black and white photograph of a drone flying in the foreground. The drone is a quadcopter with a white protective cover over its camera. In the background, there is a construction site with several large cranes and a building under construction. The scene is slightly blurred, suggesting motion or a shallow depth of field.

Thank you!

Please ask your question!